

Frequently Asked Questions about the Labor Theory of Value

<http://pandora.simons-rock.edu/~eatonak/LTV-FAQ.html>

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1.0 Introduction: What is the Labor Theory of Value (LTV)?

The LTV is the theory that market prices are attracted by prices proportional to the labor time embodied in commodities. In other words, relative prices tend towards relative labor values. The LTV is restricted to the analysis of reproducible commodities that have a use value in a capitalist society. Although the LTV is commonly associated with Classical economics, arguably neither Marx nor any first tier Classical economist accepted the LTV as a valid theory for capitalist economies.

Much of the controversy about the LTV deals with associated doctrines, particularly the doctrine that exploitation of the worker is the ultimate source of profits in a capitalist economy. David Ricardo, one of the greatest Classical economists, and Karl Marx thought that their analyses had greater applicability than the special cases in which the LTV is valid as a theory of price. [Terry Peach](#) argues that Ricardo accepted the LTV when writing the first edition of *The Principles of Political Economy and Taxation*, but not before or when modifying his book for later editions. Marx seemed to think that abstract labor time is a common substance in all commodities, while denying that prices tended to be proportional to labor values.

This FAQ is an introduction to the LTV. It's perspective is mainly that of a "Dual System" approach that has dominated Western academic economists' [discussion](#) of the LTV for the last century (when they have noticed the LTV at all). Recent developments, such as [Duncan Foley's](#) and Gerard Dumenil's "New Interpretation" and the "Temporal Single System" approach of [Guglielmo Carchedi](#), [Alan Freeman](#), [Andrew Kliman](#) and others, are, at most, treated cursorily. A more comprehensive treatment of formalizations of Marx's economic theory would discuss his third volume argument for the law of the tendency of the rate of profits to decline, the [Okishio theorem's](#) use in refuting this law, and [Andrew Kliman's](#) defense of Marx's law in a dynamic setting.

2.0 What Characteristic Features of Capitalism Provide the Setting for the LTV?

The LTV applies to commodities produced and sold in capitalist societies. Commodities have both use values and exchange values. Value, in Marx's view, is abstract labor time. The accumulation of exchange values as an end in itself distinguishes capitalism from simple commodity production.

2.1 What is a commodity?

A commodity is...an object outside us, a thing that by its properties satisfies human wants of some sort or another... [\(Karl Marx 1867, Chapter I, Section 1\)](#)

Marx later adds that a commodity implies exchange; it is regarded as containing exchange-value. Thus a person who produces something to use themselves is producing a use-value, not a commodity:

A thing can be useful, and the product of human labor, without being a commodity. Whoever directly satisfies his wants with the produce of his own labor, creates, indeed, use-values, but not commodities. In order to produce the latter, he must not only produce use-values, but values for others, social use-values. [\(Karl Marx 1867, Chapter I, Section 1\)](#)

2.2 What are "use values?"

Every useful thing...is an assemblage of many properties, and may therefore be of use in various ways...The utility of a thing makes it a use-value...When treating of use-value, we always assume to be dealing with definite quantities, such as dozens of watches, yards of linen, or tons of iron. [\(Karl Marx 1867, Chapter I, Section 1\)](#)

Utility then is not the measure of exchangeable value, although it is absolutely essential to it. If a commodity were in no way useful, - in other words, if it could in no way contribute to our gratification, - it would be destitute of exchangeable value, however scarce it might be, or whatever quantity of labor might be necessary to procure it. [\(David Ricardo 1821, Chapter I, Section I\)](#)

Note that for Marx and arguably for Ricardo, use values are qualitative, not quantitative measures along a single dimension.

2.3 What is "exchange value?"

The exchange value of a commodity is "the power of purchasing other goods which the possession of that object conveys" ([David Ricardo](#)).¹ Exchange value "presents itself as a quantitative relation, as the proportion in which values in use of one sort are exchanged for those of another sort" ([Karl Marx 1867, Chapter I, Section 1](#)). In other words, exchange value initially presents itself as the relative prices of a commodity.

2.4 What is meant by "concrete labor" and "abstract labor?"

In Marx's view, just as a commodity is "a complex of two things - use-value and exchange-value," so labor has a two-fold nature under capitalism. Just as the physical properties of a commodity determine its use value, so concrete labor activities are required to produce commodities. Marx mentions the labor of the joiner, mason, and spinner, for example.

The rate at which a coat and linen exchange in the market shows the coat and linen to be different quantities of some common substance. The coat and linen are qualitatively different use values, and so they are produced by qualitatively different kinds of concrete labor activities - tailoring and weaving:

Just as, therefore, in viewing the coat and linen as values, we abstract from their different use-values, so it is with the labour represented by these values: we disregard the difference between its useful forms, weaving and tailoring. As the use-values, coat and linen, are combinations of special productive activities with cloth and yarn, while the the values, coat and linen, are, on the other hand, mere homogeneous congelations of undifferentiated labour, so the labour embodied in the latter values does not count by virtue of its productive relation to cloth and yarn, but only as being expenditure of human labor-power. Tailoring and weaving are necessary factors in the creation of the use-values, coat and linen, precisely because these two kinds of labor are of different qualities; but only in so far as abstraction is made from their special qualities, only in so far as both possess the same quality of being human labor, do tailoring and weaving form the substance of the values of the same articles. ([Karl Marx 1867, Chapter I, Section 2](#)).

Abstract labor is this homogeneous human labor in which abstraction has been made from concrete labor activities.

¹ Here Ricardo is citing [Adam Smith](#).

2.5 What is "commodity fetishism?"

The confusion of social relationships between people as relationships between things. Marx's account of commodity fetishism in *Capital* is related to his views on alienation in his earlier work.

The reproduction of a capitalist society requires many concrete activities to be performed in parallel in different industries, as in this simple [example](#). There is a certain allocation among industries of all of the labor available to a society when it reproduces itself smoothly. But this allocation is not apparent to the workers or the capitalists. Nor can the participants know if labor is allocated properly until the capitalists try to sell the products on the market. Only then can the capitalists determine if the labor they employed was socially necessary.

Since the producers do not come into social contact with each other until they exchange their products, the specific social character of each producer's labour does not show itself except in the act of exchange. In other words, the labour of the individual asserts itself as a part of the labour of society, only by means of the relations which the act of exchange establishes directly between the products, and indirectly, through them, between the producers. To the latter, therefore, the relations connecting the labour of one individual with that of the rest appear, not as direct social relations between individuals at work, but as what they really are, material relations between persons and social relations between things. ([Karl Marx 1867, Chapter I, Section 4](#)).

Commodity fetishism prevents socially necessary abstract labor from being perceived as the substance of value. Connections between Marx's views on commodity fetishism and value theory can be seen by comparing and contrasting the above explanation of commodity fetishism with his [11 July 1868 letter to Kugelmann](#):

[E]ven if there were no chapter on 'value' in my book, the analysis of the real relationships which I give would contain the proof of the real value relation. The nonsense about proving the concept of value arises from complete ignorance both of the subject dealt with and of the method of science. Every child knows that a country which ceased to work, I will not say for a year, but for a few weeks would die. Every child knows too, that the mass of products corresponding to the different needs require different and quantitatively determined means of the total labour of society. That this necessity of distributing social labour in definite proportions cannot be done away with by the *particular form* of social production but can only change *form it assumes*, is self evident. No natural laws can be done away with. What can change, in changing historical circumstances, is the *form* in which these laws operate. And the form which this proportional division of labour operates, in a state of society where the interconnection of social labour is manifested in the *private exchange* of the individual products of labour, is precisely the *exchange value* of these products. The science consists precisely in working out *how* the law of value operates. So that if one wanted at the very beginning to 'explain' all the phenomena which apparently contradicted the law, one would have to give the science *before* the science.

2.6 How does simple commodity production differ from capitalism?

Marx uses the formula C-M-C to describe the exchange of commodities under simple commodity production, also called petty commodity production. C denotes commodities and M denotes money. The formula C-M-C shows that money, that is, a commodity's value form, intervenes in a process in which one set of use values is traded for another.

Marx contrasts the role of money in simple commodity production with its role in the formula M-C-M. Here one finds

the transformation of money into commodities, and the change of commodities back again into money; or buying in order to sell. Money that circulates in the latter manner [M-C-M] is thereby transformed into, becomes capital, and is already potentially capital. ([Karl Marx 1867, Chapter IV](#))

The merchant who buys a commodity in order to sell it again is trying to obtain more money than with which he began. Money, or more generally, the accumulation of capital has become a motive in itself. But what a trader gains in monetary value in a trade another trader loses. So gains from trade cannot be generalized to the world as a whole as an explanation for the source of surplus value. Yet capitalism is a system in which capitalists systematically make profits from the buying and selling of commodities, including the buying of inputs into production and the selling of produced goods. How, in principle, is this possible? The Marxist theory of [exploitation](#) answers this question.

Marx later expands M-C-M to M-C...P...C'-M' where money is used to purchase commodities consisting of the means of production and labor power. The capitalist removes these commodities from the market and they enter the production process P. The products re-enter the market as commodities which sell for money.

2.7 What are the means of production?

The tools, raw materials, and technology required to produce a commodity. The means of production are privately owned in a capitalist society.

3.0 What Are Labor Values?

The labor value of a commodity is amount of socially-necessary abstract labor time embodied in that commodity.

3.1 Can you give an example?

Consider a very simple capitalist economy with a yearly cycle of production in which iron is produced from inputs of iron and labor. The inputs are purchased at the beginning of the year, and the produced iron is available at the end of the year. All the iron input is used up in producing the iron outputs. Suppose quantities flows are as shown in the following table:

Example 3.1 Quantity Flows			
28 t. iron	&	56 workers	-> 56 t. iron

One Method of Calculating Labor Values. Twenty eight tons of the produced iron can be used to replace the iron input into this production process, leaving a surplus of 28 tons. In net terms, 56 workers produce a net output of 28 tons, or the net output is produced by 2 workers per ton iron. Thus, the labor value of a ton of iron is 2 person-years.

Another Method of Calculating Labor Values. Imagine that this technique has been used forever in the past. So the inputs of iron used in the current year, say 1997, were produced by inputs of labor and iron acquired in the previous year. This intellectual construction can be extended backwards indefinitely:

Example 3.1 Dated Quantity Flows		
Year	Iron Input/Output	Labor Input
1997	56 t. iron	
1996	28 t. iron	56 person-years
1995	14 t. iron	28 person-years
1994	7 t. iron	14 person-years
1993	3.5 t. iron	7 person-years
	.	
	.	
	.	
	Sum:	$56 (1 + 1/2 + 1/4 + \dots) = 112$ person-years

Notice that the above table shows the same proportions of inputs and outputs for each year's production process, as is to be expected if this technique is repeated year by year. With the exception of 1997, the output of iron for each year is used as input for the following year. The production of iron in this example with iron and labor in one year can be reduced to production with dated inputs consisting solely of labor. Fifty six tons iron embodies 112 person-years of labor. That is, one ton iron embodies two person-years.

3.2 Can you explain a more complicated example?

Consider another simple capitalist economy. This economy produces wheat and iron in a yearly cycle of production. Inputs and outputs are shown in the following table:

Example 3.2 Quantity Flows					
74 qr. wheat	&	37 t. iron	&	592 workers	-> 592 qr. wheat
18 qr. wheat	&	3 t. iron	&	48 workers	-> 48 t. iron

These quantity flows can be broken down into two subsystems, one for wheat and another for iron, as shown in the following tables:

Example 3.2 Wheat Subsystem					
$73 \frac{9}{17}$ qr. wheat	&	$36 \frac{13}{17}$ t. iron	&	$588 \frac{4}{17}$ workers	-> $588 \frac{4}{17}$ qr. wheat
$14 \frac{12}{17}$ qr. wheat	&	$2 \frac{23}{51}$ t. iron	&	$39 \frac{11}{51}$ workers	-> $39 \frac{11}{51}$ t. iron

Example 3.2 Iron Subsystem					
$\frac{8}{17}$ qr. wheat	&	$\frac{4}{17}$ t. iron	&	$3 \frac{13}{17}$ workers	-> $3 \frac{13}{17}$ qr. wheat
$3 \frac{5}{17}$ qr. wheat	&	$\frac{28}{51}$ t. iron	&	$8 \frac{40}{51}$ workers	-> $8 \frac{40}{51}$ t. iron

The proportions of inputs and outputs in the wheat industry is the same in each subsystem and the overall economy. These proportions identify the production process used in the wheat industry. The proportions of inputs and outputs in the iron industry are also unchanged between the subsystems and the overall economy. The subsystems show a conceptual division of the given quantity flows across subsystems.

$39 \frac{11}{51}$ tons iron are produced and productively consumed in the wheat subsystem. The net output of the wheat subsystem consists of 500 quarters wheat alone. In effect, the wheat subsystem is a vertically-integrated industry for producing wheat. $627 \frac{23}{51}$ person-years labor is the only non-reproduced input in the wheat subsystem. Hence $627 \frac{23}{51}$ person-years are embodied in 500 quarters wheat, or the labor value of wheat is $1 \frac{13}{51}$ person-years per quarter.

The iron subsystem shows a vertically-integrated industry for producing a net output of 8 tons iron with inputs of $12 \frac{28}{51}$ person years. The labor value of iron is $1 \frac{29}{51}$ person-years per ton. Notice that the ratio of the labor value of iron to the labor value of wheat, $1 \frac{1}{4}$ quarters per ton, has the dimensions of a relative price.

One could also calculate labor values for this example by a reduction of all inputs to dated labor flows.

3.3 What units are labor values measured in?

The answer would seem to be obviously hours or person-years, which is indeed the unit David Ricardo used to measure labor values. But Marx measured value in monetary units, such as British pounds. In some of his most difficult passages, Marx argues that money is the "universal equivalent form" of "value in general." For example,

The first chief function of money is to supply commodities with the material for the expression of their values, or to represent their values as magnitudes of the same denomination, qualitatively equal, and quantitatively comparable. It thus serves as a *universal measure of value*...It is not money that renders commodities commensurable. Just the contrary. It is because all commodities, as values, are realised human labour, and therefore commensurable, that their values can be measured by one and the same special commodity, and the latter be converted into the common measure of their values, i.e., into money. Money as a measure of value, is the phenomenal form that must of necessity be assumed by that measure of value which is immanent in commodities, labour-time. ([Karl Marx 1867, Chapter III, Section 1](#)).

The opening chapters of *Capital* have been a continual subject of debate among Marxists. For example, some have located an initial statement and solution of the [transformation problem](#) here, rather than in the third volume. The opening chapters of Volume 1 provide suggestive passages along these lines:

The price-form, however, is not only compatible with the possibility of a quantitative incongruity between magnitude of value and price, i.e., between the former and its expression in money, but it may also conceal a qualitative inconsistency, so much so, that, although money is nothing but the value-form of commodities, price ceases altogether to express value. Objects that in themselves are no commodities, such as conscience, honour, etc., are capable of being offered for sale by their holders, and of thus acquiring, through their price, the form of commodities. Hence an object may have a price without having value. ([Karl Marx 1867, Chapter III, Section 1](#)).

Notice that besides stating that the expression of value in money offers the possibility of a divergence between value and price, Marx here extends the LTV to cover commodities that are not and cannot be produced by labor.

3.4 How are labor values calculated for processes exhibiting joint production?

This question is not really asked frequently, but it's answer is helpful in understanding issues associated with [fixed capital \(machinery\)](#) and [natural resources](#).

Joint production occurs when a single production process has outputs of two or more commodities, such as wool and mutton. A process can no longer be identified with an industry; more than one process may be simultaneously in use for producing the same set

of outputs. The analysis of the choice of technique must be handled simultaneously with the analysis of joint production.

Many analytical difficulties arise in the study of joint production, and some are associated with the determination of labor values. In general, one cannot calculate labor values for joint production by a reduction of inputs to dated labor flows. Nor can one necessarily calculate the labor value of a given commodity by creating a subsystem with the same processes used in the chosen technique, but a net output consisting solely of the desired commodity. One can, however, create two subsystems whose net output differs only by a quantity of the given commodity. The difference in labor inputs between these two subsystems can be said to be the labor embodied in that quantity of the given commodity. The labor value calculated by this method may be negative. [Michio Morishima](#) has proposed an alternative method for calculating labor values for systems with joint production. He proposes that the labor value of a commodity bundle should be the minimum amount of person hours needed to produce the desired quantities of commodity net.

3.5 How is the use of capital goods accounted for in calculating labor values?

Marx considers capital to consist of [variable capital](#) and [constant capital](#). Those who might ask this question are probably thinking of the material components comprising constant capital alone. These material components can be divided into circulating capital and fixed capital. That portion of circulating capital that is also constant capital consists of those commodities, such as raw materials and semi-finished goods, that are totally used up in the (yearly) cycle of production and must be reproduced. Fixed capital consists of commodities that last for several production cycles, namely machinery and tools. Marx criticized the classical economists for frequently confusing circulating and variable capital.

Labor values are easily calculated for production processes that use circulating capital. This was demonstrated above with [Example 3.1](#) and [Example 3.2](#). Fixed capital can be correctly analyzed as a type of [joint production](#). The outputs of a production process using a machine consist of whatever commodity is usually thought of as being produced by that process and a machine one year older. The labor value of the commodity can be defined as either the minimum amount of labor needed to produce it or the labor inputs into a system with just that commodity as net output and the machine being used for the same length of time that it is actually used in the economy.

3.6 Can you illustrate how to calculate labor values with fixed capital by an example?

(This example is taken from [Ian Steedman](#).)

Consider a simple capitalist economy in which wheat is produced with the aid of a machine that can be used for two years. The machine is produced by a production process using

inputs of wheat and labor. The production processes in use in this economy, each of which require a year to complete, are shown in the following table:

Example 3.3 Quantity Flows						
84 qr. wheat	&	0 machines	&	84 workers	->	84 new machines
1,372 qr. wheat	&	84 new machines	&	840 workers	->	2,464 qr. wheat & 84 old machines
84 qr. wheat	&	84 old machines	&	840 workers	->	840 qr. wheat

Assume these production processes exhibit constant returns to scale.

In this example, 1,764 workers produce a net output of 1,764 qr. wheat net. So the labor value of wheat seems to be 1 person-year per quarter.

Choice of Technique. Could a capitalist economy operate with the technique shown above? Assume that a one-year old machine can be disposed of without cost. Then the capitalists could choose to produce the same net output with two production processes operating at the levels shown in the following table:

Example 3.3 Quantity Flows With A Machine Only Used For A Year						
147 qr. wheat	&	0 machines	&	147 workers	->	147 new machines
2,401 qr. wheat	&	147 new machines	&	1,470 workers	->	4,312 qr. wheat

The proportions of inputs to outputs are unchanged from the corresponding processes when the machine is used for two years. Thus the processes for producing a new machine and for producing wheat with a new machine are unchanged between these two techniques, merely operated at different scales.

When the year-old machine is discarded, 1,617 workers can produce the same net output of 1,764 qr. wheat. So the efficient technique that would be chosen by a planned socialist state would result in 11/12 person-years being embodied in a quarter wheat. That is, more output can be produced with the same labor inputs by discarding the machine after it is used for only a year. **But a physically inefficient technique can be adopted in a capitalist economy!**

A full analysis of the choice of technique can only be performed by considering prices of production. Accordingly, consider the following set of prices:

Possible Prices in Example 3.3	
Price of 1 qr. wheat:	\$1
Price of 1 new machine:	$\$1 \cdot 79/84 = \1.94
Price of 1 old machine:	\$0
Wage for one person-year:	$\$311/504 = \0.62
Rate of profits:	20%

These prices will result in revenues covering costs at the going rate of profits for the two processes in the technique in which the one-year old machine is junked:

$$(147 \times \$1 + 147 \times \$0.62)(1 + 0.20) = 147 \times \$1.94$$

$$(2,401 \times \$1 + 147 \times \$1.94 + 1,470 \times \$0.62)(1 + 0.20) = 4,312 \times \$1$$

Firms producing wheat at these prices will have no incentive to enter into machine manufacturing. Likewise, firms producing machines will have no incentive to remove their capital from machine manufacturing and enter into wheat production. But will a firm producing wheat have a monetary incentive to put some of their labor force to work producing wheat with a one-year old machine? Consider the costs and revenues at these prices for adding this new process:

$$\text{Cost} = (84 \times \$1 + 84 \times \$0 + 840 \times \$0.62)(1 + 0.20) = \$726$$

$$\text{Revenue} = 840 \times \$1 = \$840$$

So extra profits will be earned by using the machine for its full lifetime. Thus, a capitalist economy can adopt the inefficient technique in which the machine is run for two years and in which one person-year is embodied in each quarter wheat produced.

Labor Values of Machines. Now that one has seen that the technique in which machines are operated for two years can be adopted, we can consider the labor embodied in one and two year old machines. Suppose the processes in this technique are operated at the levels shown in the following table:

Example 3.3 Quantity Flows for Producing New Machines							
172 qr. wheat	&	0 machines	&	172 workers	->	172 new machines	
1,437 1/3 qr. wheat	&	88 new machines	&	880 workers	->	2,581 1/3 qr. wheat	& 88 old machines
88 qr. wheat	&	88 old machines	&	880 workers	->	880 qr. wheat	

The net output here is 1,764 qr. wheat and 84 new machines. We have seen that 1,764 of the 1,932 workers are required to produce the wheat. Hence the remaining 168 workers produce the 84 new machines. In other words, the labor value of a new machine is 2 workers per new machine.

We can also consider levels of operation of the production processes such that net output consists of the same quantity of wheat and additional old machines. In this example, this increase in net output is brought about by changing the scale of the processes such that total labor inputs **decrease**:

Example 3.3 Quantity Flows for Producing Old Machines							
120 qr. wheat	&	0 machines	&	120 workers	->	120 new machines	
1,960 qr. wheat	&	120 new machines	&	1,200 workers	->	3,520 qr. wheat	& 120 old machines
36 qr. wheat	&	36 old machines	&	360 workers	->	360 qr. wheat	

Net output, which is produced by 1,680 workers consists of 1,764 qr. wheat and 84 old machines. 1,764 workers are required to produce 1,764 qr. wheat. So -84 workers are required to produce 84 old machines. Hence, the labor value of old machines is -1 worker per old machine. **Labor values can be negative** when they are calculated by this method.

Conclusions. This example demonstrates that labor values can be determined for capitalist economies that make use of fixed capital (that is, actually existing capitalist economies). If one adopts a definition of labor values such that Marx is correct in treating the labor embodied in a commodity basket as the sum of the labor values of the individual commodities, the following problems arise:

- The analysis of labor values requires a prior analysis of the choice of technique, which, contrary to Marx's theory, requires a consideration of phenomena on the level of [prices of production](#).
- The labor value of some commodities can be negative.
- The rate of profits can be positive even when workers are not exploited.

Alternatively, one can define the labor value of a bundle of goods as the minimum number of person-hours needed to produce that bundle as the net output. With this definition, the [fundamental theorem of Marxism](#), that the rate of profits is positive if and only if labor is exploited, remains true. Furthermore, although the calculation of labor values requires an analysis of the choice of technique, this analysis need only consider the physical data of available production processes; the calculation of labor values does not require a prior determination of [prices of production](#). Marx's method of summing labor values, however, is invalid under this approach.

3.7 How is the use of natural resources accounted for in calculating labor values?

Durable non-produced means of production, such as land of a given fertility, are a special case of [joint production](#). Land can be regarded as simultaneously an input and an output to certain production processes. Accordingly, the labor value of a commodity produced with the aid of natural resources (either directly or indirectly) is determined by the labor time required to increase the net output of society by one more unit of that commodity. This calculation will require an analysis of marginal land.

Exhaustible resources, which can be used as inputs but are not outputs of any production processes, present difficulties for the LTV. Examples include coal, oil, and various sorts of metallic ores. The Classical economists analyzed exhaustible resources by analogy to land. They generally ignored in their value theory that fertile mines would eventually be depleted. This procedure might be defended on the grounds that the time scale in which mines are depleted is longer than the time period for which labor values are calculated. This defense seems weak today when ecological issues are of concern.

3.8 Not all workers have the same abilities. How can labor values be meaningful?

Those seriously interested in this question should consider all of Chapter 1 of [Marx \(1867\)](#), especially Marx's distinction between [concrete and abstract labor](#). Part of the difficulty in understanding Marx's notion of abstract labor is that he thought his theory reflected the topsy-turvy world of capitalism:

When I state that coats or boots stand in a relation to linen, because it is the universal incarnation of abstract human labor, the absurdity of the statement is self-evident. Nevertheless, when the producers of coats and boots compare these articles with linen, or, what is the same thing with gold or silver, as the universal equivalent, they express the relation between their own private labour and the collective labour of society in the same absurd form. ([Karl Marx 1867, Chapter I, Section 4](#))

[Robert Paul Wolff](#) argues Marx chose the writing style he did, with all its attendant difficulties, to reflect his theory of the absurdities of capitalism.

Marx claims that different labors are equated when products of labor exchange on capitalist markets:

Hence, when we bring the products of our labour into relation with each other as values, it is not because we see in these articles the material receptacles of homogeneous human labour. Quite the contrary; whenever, by an exchange, we equate as values our different products, by that very act, we also equate, as human labour, the different kinds of labour expended upon them. We are not aware of this, nevertheless we do it. Value, therefore, does not stalk about with a label describing what it is. ([Karl Marx 1867, Chapter I, Section 4](#)).

Perhaps Marx is more clear on this point in [The Poverty of Philosophy](#):

Does labour time, as the measure of value, suppose at least that the days are *equivalent*, and that one man's day is worth as much as another's? No.

Let us suppose for a moment that a jeweller's day is equivalent to three days of a weaver; the fact remains that any change in the value of jewels relative to that of woven materials, unless it be the transitory result of the fluctuation of demand and supply, must have as its cause a reduction or an increase in the labour time expended in the production of one or the other. If three working days of different workers be related to one another in the ratio 1:2:3, then a change in the relative value of their products will be a change in the same proportion of 1:2:3. Thus values can be measured by labour time, in spite of the inequality of value of different working days; but to apply such a measure we must have a comparative scale of the different working days: it is competition that sets up this scale.

Is your hour's labour worth mine? That is a question which is decided by competition. ([Karl Marx 1955, Chapter I, Section 2](#)).

The Classical economists Adam Smith and David Ricardo also thought different concrete labors could be reduced to a single measure of homogeneous labor. They relied on the supposed empirical fact of the stability of relative wages:

In speaking, however, of labour, as being the foundation of all value, and the relative quantity of labour as almost exclusively determining the relative value of commodities, I must not be supposed to be inattentive to the different qualities of labour, and the difficulty of comparing an hour's or a day's labour, in one employment, with the same duration of labour in another. The estimation in which different qualities of labour are held, comes soon to be adjusted in the market with sufficient precision for all practical purposes, and depends much on the comparative skill of the labourer, and intensity of the labour performed. The scale, when once formed, is liable to little variation. If a day's labour of a working jeweller be more valuable than a day's labour of a common labourer, it has long ago been adjusted, and placed in its proper position in the scale of value. ([David Ricardo 1821, Chapter I, Section II](#))²

Ricardo quotes Adam Smith with approval:

But though labour be the real measure of the exchangeable value of all commodities, it is not that by which their value is commonly estimated. It is often difficult to ascertain the proportion between two different quantities of labour. The time spent in two different sorts of work will not alone determine this proportion. The different degrees of hardship endured, and of ingenuity exercised, must likewise be taken into account. There may be more labour in an hour's hard work, than in two hours easy business; or, an hour's application to a trade which it costs ten years' labour to learn, than in a month's industry at an ordinary and obvious employment. But it is not easy to find any accurate measure, either of hardship or ingenuity. In exchanging, indeed, the different productions of different sorts of labour for one another, some allowance is commonly made for both. It is adjusted, however, not by any accurate measure, but by the higgling and bargaining of the market, according to that sort of rough equality, which though not exact, is sufficient for carrying on the business of common life. ([Smith 1776, Book I, Chapter V](#))

(See also [Smith 1776, Book I, Chapter X](#).) It is interesting to note similarities between the Marx and Smith quotes above.

Anyways, [Ian Steedman](#) has shown that the [fundamental theorem of Marxism](#) is consistent with heterogeneous labor activities not reduced to a single measure of homogeneous labor.

² Originally this was attributed to Section IV, which was incorrect.

3.9 Doesn't the LTV assert it is desirable for workers to become more lazy so the goods they produce would increase in value?

No.

A country prospers not by increasing the labor value of a given output, but by increasing the use values that can be produced by the labor force. The "necessaries and conveniences of life," as Adam Smith has it, are increased by *decreasing* the labor values of commodities.

Furthermore, capitalists have no incentive to increase labor values as such. Capitalists increase their profit by increasing surplus value. Surplus value is increased by increasing relative and absolute surplus value. Absolute and relative surplus value is increased by increasing the rate of exploitation of the workers. *Decreasing* the labor values of commodities results in an increase in relative surplus value.

Finally, Marx explicitly denied the premise of this question:

Some people might think that if the value of a commodity is determined by the quantity of labour spent on it, the more idle and unskillful the labourer, the more valuable would his commodity be, because more time would be required in its production. The labour, however, that forms the substance of value, is homogeneous human labour, expenditure of one uniform labour-power. The total labour-power of society, which is embodied in the sum total of the values of all commodities produced by that society, counts here as one homogeneous mass of human labour-power, composed though it be of innumerable individual units. Each of these units is the same as any other, so far as it has the character of the average labour-power of society, and takes effect as such; that is, so far as it requires for producing a commodity, no more time than is needed on an average, no more time than is socially necessary. The labour-time socially necessary is that required to produce an article under the normal conditions of production, and with the average degree of skill and intensity prevalent at the time... We see then that that which determines the magnitude of the value of any article is the amount of labour socially necessary, or the labour-time socially necessary for its production. ([Karl Marx 1867, Chapter I, Section 1](#)).

3.10 The cost of a good includes more than the cost of labor. Doesn't this observation invalidate the LTV?

The labor value of a commodity is not the exchange value of the labor that is used to produce that commodity. Rather, the labor value of a commodity is the time required to produce a commodity, including the labor time needed to produce the capital goods with which that commodity is produced.

The price at which a good is typically sold exceeds the wages which would cover the labor time required to produce the good. This excess reflects a going rate of profit. This observation is no refutation of the LTV, but exactly what the LTV was used by Marx to

explain, namely the [exploitation](#) of the worker under capitalism. Interestingly enough, Adam Smith makes a related observation:

As soon as stock has accumulated in the hands of particular persons, some of them will naturally employ it in setting to work industrious people, whom they will supply with materials and subsistence, in order to make a profit by the sale of their work, or by what their labour adds to the value of the materials...The value which the workmen add to the materials, therefore, resolves itself in this case into two parts, of which one pays their wages, the other the profits of their employer upon the whole stock of materials and wages which he advanced. ([Smith 1776, Book I, Chapter VI](#))

Perhaps, those who ask this question are objecting to the nonsymmetrical treatment of factors in the LTV. Bourgeois economists typically explain the incomes received by labor, capital, and land by an [incorrect](#) theory in which the forces of supply and demand operating in each factor market determine each factor's return. Although the LTV does include an analysis of [capital](#) and [land](#), factors are not treated symmetrically in the LTV. For example, profits are explained as a residual in Classical economics.

3.11 How can past costs determine value?

As noted [above](#), labor values are not costs. The [first example above](#) illustrates that labor values can be thought of as sums of past labor inputs or as quantities reflecting production processes occurring simultaneously. Labor values are not measures of past decisions under this latter conception. Marx was aware of this distinction:

Since the continuous, constantly repeated process of production is, at the same time, a process of reproduction, it is equally dependent on the *coexisting labour* which produces the various phases of the product simultaneously, while the product is passing through metamorphosis from one phase to another. [Raw] cotton, yarn, fabric, are not only produced one after the other and from one another, but they are produced and reproduced *simultaneously*, alongside one another. What appears as the effect of antecedent labour, if one considers the production process of the individual commodity, presents itself at the same time as the effect of coexisting labour, if one considers the *reproduction process* of the commodity, that is, if one considers this production process in its continuous motion and in the entirety of its conditions, and not merely an isolated action or a limited part of it. There exists not only a cycle comprising various phases, but all the phases are simultaneously produced in the various spheres and branches of production. If the same peasant just plants flax, then spins it, then weaves it, these operations are performed in succession, but not simultaneously as the mode of production based on the division of labour within society presupposes. ([Karl Marx 1971, Chapter XXI, Section 3.b](#))

This idea is also insightfully presented in [Volume 2](#) of *Capital*.

4.0 What Is Exploitation?

Workers are exploited under capitalism when the commodities which they purchase with their wages embody less labor-time than they expend in earning their wages.

4.1 What is the Fundamental Theorem of Marxism?

The rate of profit is positive in the system of [prices of production](#) if and only if some workers are exploited. [Michio Morishima](#) stated and proved this mathematical theorem in his interpretation of Marx's economics. It is true, with an appropriate definition of [labor values](#), under a wide range of assumptions, including the existence of joint production, of production processes using circulating and fixed capital, of natural resources used in production, and of heterogeneous labor not reducible to a single dimension of abstract labor time.

4.2 If all commodities sold at their value, how would the capitalists be able to exploit the workers?

Marx assumed the assumptions under which the LTV holds just so as to be able to pose and answer this question in the first volume of *Capital*.

Given Marx's definition of labor values, the labor value of output is the sum of the direct labor hours used to produce that output and the labor embodied in the means of production the laborers work up into the output. Since the labor value of the means of production are transferred unchanged to the output, Marx calls the means of production [constant capital](#), and their labor values are denoted by C . According to Marx, workers do not sell their labor time, but the ability to work under the capitalists' direction. Marx called this commodity [labour power](#). The use value of labour power is the labor time for which laborers work. Since this commodity produces a greater value than it costs, Marx called labor power [variable capital](#) when viewed from the standpoint of the capitalist production process. The labor value of variable capital is denoted by V . [Surplus value](#), denoted by S , results from the laborers working longer hours than needed to reproduce $C + V$. The labor value of output, in Marx's labor accounting scheme, is $C + V + S$.

Exploitation is possible under capitalism because the capitalists purchase variable capital with part of their capital. When workers labor under the capitalists' direction, the capitalists do their best to ensure the use value of labor power is a greater amount of time than the labor value of labor power.

4.3 If all exchanges are freely made and no one is forced, how is exploitation possible?

This objection seems to miss the point of Marx's account of exploitation. Marx acknowledged that market exchanges are freely made:

This sphere,...within whose boundaries the sale and purchase of labour-power goes on, is in fact a very Eden of the innate rights of man. There alone rule Freedom, Equality, Property, and Bentham. Freedom, because both buyer and seller of a commodity, say of labour-power, are constrained only by their own free will. They contract as free agents, and the agreement they come to, is but the form in which they give expression to their common will. Equality, because each enters into relation with the other, as with a simple owner of commodities, and they exchange equivalent for equivalent. Property, because each disposes only of what is his own. And Bentham, because each looks only to himself. The only force that brings them together and puts them in relation with each other, is the selfishness, the gain and the private interests of each. Each looks to himself only, and no one troubles himself about the rest, and just because they do so, do they all, in accordance with the pre-established harmony of things, or under the auspices of an all-shrewd providence, work together to their mutual advantage, for the common weal and in the interest of all. ([Karl Marx 1867, Chapter VI](#))

Perhaps Marx intended that last sentence ironically. But consider the following quote, in which Marx describes the gains from trade:

So far as regards use-values, it is clear that both parties may gain some advantage. Both part with goods that, as use-values, are of no service to them, and receive others that they can make use of. And there may also be a further gain. A, who sells wine and buys corn, possibly produces more wine, with given labour time than farmer B could, and B, on the other hand, more corn than wine-grower A could. A, therefore, may get, for the same exchange value, more corn, and B more wine, than each would respectively get without any exchange by producing his own corn and wine. With reference, therefore, to use-value, there is good ground for saying that 'exchange is a transaction by which both sides gain.' ([Karl Marx 1867, Chapter V](#))

So it would seem that proponents of the objection to the Marxist account of exploitation need to consider how Marx could have found these ideas consistent with his theory. Note that these quotes describe only the exchange of use-values. They do not characterize the unity of production and circulation under capitalist institutions.

4.4 What is labor power?

By labour-power or capacity for labour is to be understood the aggregate of those mental and physical capabilities existing in a human being, which he exercises whenever he produces a use-value of any description. ([Karl Marx 1867, Chapter VI](#)).

According to Marx, labor power is available on the market as a commodity that the capitalist can buy only if the worker has the "double freedom" of being free to sell his labor-power and being free from owning land or produced means of production. If the worker used means of production that he owned, he would sell the products of his labor, not his labor power.

The labor embodied in labor power is the labor value of the means of subsistence which the worker requires to reproduce his labor power. In other words, the value of labor power is the labor value of "necessary consumption" used to maintain the workers. The use value of labor power is the labor which the capitalist can get out of the worker. The difference between the number of hours which the worker works and the labor value of labor power is the source of profit under capitalism.

4.5 What is constant capital?

That part of capital then, which is represented by the means of production, by the raw material, auxiliary material and the instruments of labour, does not, in the process of production, undergo any quantitative alteration of value. I therefore call it the constant part of capital, or, more shortly *constant capital*. ([Karl Marx 1867, Chapter VIII](#)).

4.6 What is variable capital?

...that part of capital, represented by [labor power](#), does in the process of production, undergo an alteration of value. It both reproduces the equivalent of its own value, and also produces an excess, a surplus-value, which may itself vary, may be more or less according to circumstances. That part of capital is continually being transformed from a constant into a variable magnitude. I therefore call it the variable part of capital, or, shortly *variable capital*. The same elements of capital which, from the point of view of the labour process, present themselves respectively as the objective and subjective factors, as means of production and labour power, present themselves, from the point of view of the process of creating surplus value, as constant and variable capital. ([Karl Marx 1867, Chapter VIII](#)).

4.7 What is the organic composition of capital?

All capital is either [constant](#) or [variable](#). The organic composition of capital is the ratio of the labor value of constant capital to the labor value of variable capital. In other words, the organic composition of capital is the ratio of the labor value of the means of production (constant) to total wages (variable). Roughly, the organic composition of capital is a measure of capital intensity.

According to Marx, [surplus value](#) can only be realized through exploitation of variable capital; therefore, a change in the organic composition of capital has an effect on working hours and wages.

4.8 What is surplus value?

Surplus value is the difference between the number of hours the laborer works and the amount of labor time necessary to maintain the worker. In other words, it is the difference between the worker's use value and the labor value of variable capital.

4.9 What is the rate of surplus value, also known as the rate of exploitation?

The rate of surplus value is the ratio of surplus value to the value of variable capital, or S/V . It can also be thought of as the ratio of surplus labor to necessary labor, where necessary labor is the labor needed to reproduce the means of production used up in a yearly production cycle and the consumption goods which support the workers. A third way of thinking of the rate of surplus value is as the ratio of the labor time for which the capitalists do not pay to the paid labor time:

The rate of surplus-value is therefore an exact expression for the degree of exploitation of labour-power by capital, or of the labourer by the capitalist. ([Karl Marx 1867, Chapter IX](#))

Therefore the rate of surplus value is also called the rate of exploitation.

4.10 How can the capitalists increase absolute surplus value?

The surplus-value produced by prolongation of the working day, I call *absolute surplus value*. On the other hand, the surplus-value arising from the curtailment of the necessary labour-time, and from the corresponding alteration in the respective lengths of the two components of the working day, I call *relative surplus value*. ([Karl Marx 1867, Chapter XII](#)).

Absolute surplus value is increased by increasing surplus value for a given labor value of variable capital. That is, absolute surplus value is increased by making laborers work for more hours per day, more weeks per year, etc.

4.11 How can the capitalists increase relative surplus value?

Marx's definition of relative surplus value was given above along with his definition of [absolute surplus value](#). Relative surplus value is increased by a reduction in the labor value of variable capital for an unchanged working day ($S + V$). This reduction can be accomplished by:

- Decreasing the wage
- Increasing the intensity with which laborers work
- Reducing wasted output and output that cannot be sold because of inadequate quality
- Reducing the use of material inputs per unit output, for example, by reducing wasted inputs
- Adopting more productive machines and using existing machines more efficiently

Since surplus value is the source of profits, it is in the capitalists' interest to increase both absolute and relative surplus value. Marx provided an analysis of many historical and contemporary examples of the practical consequences of these pressures.

5.0 What Are Prices of Production?

Prices of production for a competitive^[sic] capitalist economy show the same rate of profit being earned in all industries. They are a set of prices that allow smooth reproduction of an economy in which goods and services are traded on markets.

5.1 Can you give an example?

The clearest example of "exchange-values [which] spring directly from the methods of production" ([P. Sraffa](#)) are found in a simple economy which does not produce a surplus and in which labor inputs do not explicitly appear. Consider an economy that produces three goods - wheat, iron, and pigs - with a yearly cycle of production. The wheat industry begins the year with inputs of 240 quarters wheat, 12 tons iron, and 18 pigs and uses these inputs to produce 450 quarters wheat. The iron industry uses 90 quarters wheat, 6 tons iron, and 12 pigs to produce 21 tons iron. The pig industry uses inputs of 120 quarters wheat, 3 tons iron, and 30 pigs to produce 60 pigs. This economy can be presented in tabular fashion:

Example 5.1 Quantity Flows					
240 qr. wheat	&	12 t. iron	&	18 pigs	-> 450 qr. wheat
90 qr. wheat	&	6 t. iron	&	12 pigs	-> 21 t. iron
120 qr. wheat	&	3 t. iron	&	30 pigs	-> 60 pigs

Notice that the outputs just replace the inputs. Each industry only has their own goods at the end of year. Production cannot continue without trade to redistribute the outputs among industries into the proportions needed for inputs. The exchange values which ensure replacement all round are 10 qr. wheat = 1 t. iron = 2 pigs. So prices of production for this example could be:

Example 5.1 Prices	
Price of 1 qr. wheat:	\$1
Price of 1 t. iron:	\$10
Price of 1 pig:	\$5

One could interpret this example economy as implicitly using labor in the production of commodities. Perhaps some of the wheat and pig inputs are used to feed the workers in each industry. Also note that this economy's reproduction requires triangular trade among the industries. Probably one good would be used as money.

(The example is taken from [Piero Sraffa](#).)

5.2 Can you give an example with a surplus and labor inputs?

Consider another simple economy. This economy produces wheat and iron in a yearly cycle of production. Inputs and outputs are shown in the following table:

Example 5.2 Quantity Flows					
74 qr. wheat	&	37 t. iron	&	592 workers	-> 592 qr. wheat
18 qr. wheat	&	3 t. iron	&	48 workers	-> 48 t. iron

Each industry's output must be sold at a price that covers the cost of the inputs so as to allow the reproduction of this economy. Since this is a competitive capitalist economy, the rate of profits must be constant across industries. Hence prices of production solve the following system of equations:

$$(74 p_w + 37 p_i + 592 w)(1 + r) = 592 p_w$$

$$(18 p_w + 3 p_i + 48 w)(1 + r) = 48 p_i$$

where

- p_w is the price of wheat
- p_i is the price of iron
- w is the wage
- r is the rate of profits.

This system of equations shows the capitalists advancing the wages to the workers. A different formulation would show the workers as advancing their labor power to the capitalists and being paid from the output.

There are four unknowns, but only two equations. One unknown is fixed by choosing a [numeraire](#), say the net output per worker. The other degree of freedom is typically taken to be the wage-rate of profits frontier. The location on that frontier could be given by taking either the wage or the rate of profits as given data. Prices of production for this little model economy are:

$$p_w = 320 / (255 + r)$$

$$p_i = 80 (5 + r) / (255 + r)$$

$$w = 5 (3 - r) (17 + r) / [(255 + r) (1 + r)]$$

Notice that the wage is higher for a lower rate of profits.

5.3 Why call these exchange values "prices of production," rather than "costs," "costs of production," "natural prices," or "necessary prices?"

These are all roughly equivalent terms in Classical and Marxian economics, but "prices of production" seems the least likely to mislead.

The expressions "costs" and "costs of production" seem to imply that prices of production depend merely on what must be paid for the means of production, wages, and profits. But this impression is one-sided for commodities that enter directly or indirectly into the production of all other commodities. Their prices of production depend upon their use in the production of other commodities as much as they depend upon the extent into which they

enter their own production. For instance, the price of iron in [the second example above](#) depends both on the commodities needed to produce it and on how much iron is used in producing wheat.

The Classical economists, particularly [Adam Smith](#) and [David Ricardo](#), used the expressions "natural prices" or "necessary prices." These terms are avoided here because of their association with mistaken or unclear theories in these authors.

Adam Smith had an "adding-up" theory of natural prices:

When the price of any commodity is neither more nor less than what is sufficient to pay the rent of the land, the wages of the labour, and the profits of the stock employed in raising, preparing, and bringing it to market, according to their natural rates, the commodity is then sold for what may be called its natural price. [\(Smith 1776, Book I, Chapter VII\)](#)

Smith incorrectly thought natural wages, rents, and profits could vary independently of one another. This Smithian theory is connected with Smith's mistaken belief that a rise in natural wages would cause a rise in all prices.

Ricardo assumed natural prices as equal to labor values as an aid to criticism of Smith's theory. If prices of production were equal to labor values, the rate of profit would be found from the industry producing wage-goods alone, where wage-goods are those commodities which the workers buy with their wages. Profits in the wage-good industry would be the difference between the labor embodied in wage-goods and the sum of the labor embodied in the means of production and the labor embodied in the wage-goods consumed by the workers in the wage-good industries. The rate of profits would be the ratio of the labor value of profits in the wage-good industry to the sum of the labor embodied in the means of production and the labor embodied in the wage-goods purchased by the workers producing wage-goods:

$$r = [v_1 - (c + v)] / (c + v) = [v_1 / (c + v)] - 1$$

where

- r is the rate of profits
- v_1 is the labor embodied in wage goods
- c is the labor embodied in the means of production of the wage-good industry
- v is the labor embodied in the wage goods consumed by the workers in the wage-good industry.

The use of labor as a measure of both input and output in the production of wage-goods shows the rate of profits as a ratio of physical quantities, independent of valuation. This makes it apparent that (real) wages cannot rise without a fall in the rate of profits, given technology. This conclusion, however, can be shown without the simplifying assumption, while elaborating Ricardo's analysis of the effects of a rise of wage on prices, including a fall in the prices of production of some commodities.

Although Ricardo's approach is an insightful simplification, it can mislead the unwary into confusing labor values and natural prices in Classical economics. Since Marx clearly distinguished between labor values and prices of production, his terminology is adopted here.

5.4 How are prices of production related to market prices?

The actual price at which any commodity is commonly sold is called its market price. It may either be above, or below, or exactly the same with its [price of production]. ([Smith 1776, Book I, Chapter VII](#))

The price of production,

therefore, is, as it were, the central price, to which the prices of all commodities are continually gravitating. Different accidents may sometimes keep them suspended a good deal above it, and sometimes force them down even somewhat below it. But whatever may be the obstacles which hinder them from settling in this center of repose and continuance, they are constantly tending towards it. ([Smith 1776, Book I, Chapter VII](#))

The articulation of this metaphor of prices of production acting as centers of gravitational attraction is a research question among some contemporary economists. For example, even if one does not think market prices tend toward prices of production, might the differences between market prices and prices of production be useful in analyzing investment plans?

5.5 What is the "realization problem?"

The realization problem is Marxist terminology arising in the analysis of differences between market prices and prices of production. If proportions between industries are inappropriate, some capitalists firms may find that they cannot sell all of their output at the corresponding prices of production. Or there may be a general overproduction in which all the commodities produced cannot be sold. (Marx, in contrast to some Classical economists, denied [Say's law](#). Say's law implies that persistent general overproduction (depression) is impossible.)

In either case, not all firms will receive the appropriate rate of profit for their cost structure. Consequently, capitalists will disinvest in some sectors and more heavily invest in others. This process will cease only if all firms can sell their output at prices of production. Adam Smith called this level of output the level of "effectual demand." Notice that effectual demand is a specified quantity, not a schedule relating quantities and prices.

5.6 Aren't prices of production merely Neoclassical long-run equilibrium prices?

No.

Neoclassical economics is commonly regarded as having been the dominant school of thought among Western academic economists for over a century. Although they had interesting precursors, [W. Stanley Jevons](#), [Carl Menger](#), and [Leon Walras](#) are usually thought to have initiated Neoclassical economics in the 1870s with almost simultaneous presentations of their theories. Briefly, Neoclassical economists claim to explain prices as the result of an equilibrium of Supply and Demand in all markets. The ultimate determinates of prices are technology, tastes (in the form of utility functions), and endowments. Equilibrium prices are thought to coordinate individual maximization problems.

[Alfred Marshall](#) replaced the Classical distinction between market prices and prices of production with the notion of equilibrium existing in various runs. The most important of Marshall's equilibrium concepts are short run and long run equilibrium. In short run equilibrium, agents in the economy have chosen the optimal level of operation of a given capacity. In long run equilibrium, capacity output, levels of operation, and the mix of all inputs are all choice variables. The associated set of long run equilibrium prices are known as "normal prices." Normal prices show all industries earning the same normal rate of profits in competitive conditions.

Prices of production are sufficiently close to the notion of Neoclassical long-run equilibrium prices that a critique of Neoclassical economics can be built upon an analysis of how prices of production vary with distribution. Prices of production, however, are conceptually distinct from Neoclassical long-run equilibrium prices. Some distinctions between the settings for Neoclassical normal long run equilibrium prices and Classical prices of production are outlined here. Elaborations on most of these themes can be found in the writings of Krishna Bharadwaj, [Pierangelo Garegnani](#), and Alessandro Roncaglia.

Competition. Neoclassical long run equilibrium prices and prices of production are based on different conceptions of competition. Perfect competition, according to Neoclassical economists, exists when no participant in the market has the power to change prices solely through their own actions. Each agent accepts all prices as given parameters. The Classical conception of competition, on the other hand, is merely that there are no barriers to entry or exit in a market. Consequently, there will be a tendency for differences in prices and the rate of profit to level out.

These different abstractions regarding competition are reflected in different theories for non-competitive markets. The Classical economist sees monopolistic barriers to entry as being reflected in persistent differences in rates of profits. A simple approach would be to assume given ratios between rates of profits in different industries in calculating prices of production.

Some academic economists have recently elaborated Classical theories of oligopoly. These Classical theories can be seen in the work of Joe Bain and Paolo Sylos Labini. Sylos Labini, at least, explicitly acknowledges the Classical elements in his theories.

Unemployment and clearing of the labor market. The Neoclassical conception of competitive equilibrium shows all markets clearing, including the labor market. Thus, persistent involuntary unemployment can be analyzed by Neoclassical economics only with great difficulty, if at all:

The structure of modern economics is inhospitable to the idea of persistent unemployment and is always trying to extrude it. Only the stubborn refusal of the brute fact to go away has kept the analytical problem alive. ([Robert Solow](#))

On the other hand, the quantity relationships for which prices of production are calculated are compatible with involuntary unemployment. Capacity is fully used when prices of production are realized, but this issue is separate from whether or not all workers can find a job. Since the same rate of profits is being achieved in all industries, there is no tendency for investment to change. Neoclassical economists may think there would be a tendency for wages to fall in a situation with involuntary unemployment and for firms to adopt relatively more labor-intensive techniques of production. But Classical economics was not based on substitution principles, and modern economists have vindicated the Classical analysis.

Some economists have therefore concluded that prices of production, and Classical economics more generally, provide the natural long period setting for a generalization of Keynes' economics. Heinrich Bortis, Jan Kregel, Edward Nell, and Luigi Pasinetti are some contemporary economists currently researching the integration of Classical economics and Keynes. This position is controversial. Some economists, for example, Paul Davidson, think this approach risks downplaying the existence of [non-ergodic uncertainty](#), which they think is the fundamental cause of unemployment in a monetary production economy.

Normative versus positive analysis. As shown by William Jaffe, Walras developed General Equilibrium Theory to illustrate a "realistic" utopia conforming to certain of his ideas of justice in exchange. Neoclassical economists have continued to found their analysis on the examination of the logical consistency of an utopia. Actually existing societies are analyzed by examining deviations from utopian norms.

Classical economics, on the other hand, is a matter of trying to understand how actually existing capitalist societies function. Although the Classical economists had a pre-analytical vision of essential characteristics of their societies, their analysis in itself is not explicitly normative in the same sense that Neoclassical economics is. Of course, the Classical economists were doing political economy and applied their analyses to political ends.

Production and consumption. Neoclassical economics begins with consumption. A pure exchange economy is a standard introductory model for Neoclassical economists. This is true historically for Walras, and quite frequently papers in bourgeois economics journals are restricted to consumption. For example, a [famous paper](#) dealt with a Prisoner of War (POW) economy in which no production occurs, but Red Cross packages provide a periodic flow of

consumer goods.³ Neoclassical economists have not reached a consensus on how to model production. In the opinion of the original author of this FAQ, there is no Neoclassical theory of production that is theoretically coherent, logically consistent, and empirically applicable.

Production is the focus of Classical economics and of analyses associated with prices of production. Although Marx recognized that commodities have a use value, he explicitly stated with his famous formula M-C...P...C'-M' that capitalist production is directed at profit.

Reproducibility versus the allocation of given resources. Neoclassical theory is not based on analyzing the reproducibility of an economy. Rather, Neoclassical theory is about the allocation of given goods and a one-way process beginning with scarce resources and ending with consumer goods being purchased to satisfy given tastes. Equilibrium prices are mistakenly taken as indicators of relative scarcities, not as ratios linked to a regular repetition of production processes. Classical economists put reproducibility at the center of their analysis. This emphasis predates the LTV and can be seen in [Francois Quesnay's *Tableau Economique*](#). If one wants to understand how capitalist economies have continued to expand for centuries and why this process might terminate, Classical economics seems to provide more appropriate analytical tools than Neoclassical economics.

Relationships between quantities and prices. Neoclassical economics shows quantities and prices as simultaneously and mutually determined in all markets. Tastes, technologies, and endowments are taken as given data. Calculating normal prices from these data leads to a logical inconsistency or, at least, theoretical incoherence. Walras attempted to obtain equilibrium prices for a stationary state. But, if endowments of produced means of production are given, only one of these capital goods is likely to be reproduced. The production of the others will obtain a lower rate of profit, and they will not be reproduced.

Prices of production, on the other hand, are calculated from a different set of data. The level and composition of output, that is, quantities, are taken as given. So is distribution. Either the rate of profits or the wage is given data. (If labor is taken as heterogeneous, either relative wages and the absolute level of one wage, or the rate of profits and all but one wage are given data.) Given quantities and distribution, prices of production and the remaining distributive variable can be calculated. This structure of Classical value theory allows one to analyze effects which directly change coefficients of production, such as the increasing returns that Adam Smith stated result from an increase in the extent of the market. With the prior determination of quantities,

...the theory of value will lose the all-embracing quality it assumed with the marginal method. But what will be lost in scope will certainly be gained in consistency and, we may hope, fruitfulness. ([P. Garegnani](#))

³ R. A. Radford. The Economic Organisation of a P.O.W. Camp. *Economica*; vol. 12, no. 48, Nov. 1945, pp.189-201.

5.7 How has an analysis of prices of production been used to construct a critique of bourgeois economics?

By showing that **given their own assumptions**, Neoclassical economists lack any sound conceptual basis for establishing the existence of markets, particularly factor markets, with well-behaved supply and demand relationships. Thus, Neoclassical economists have yet to show that an equilibration of supply and demand, as conceived in Neoclassical economics, can explain prices.

This critique was promoted among academic economists by the Neo-Ricardian or Sraffian school in what has become known as the [Cambridge Capital Controversy \(Harcourt 1972\)](#). Neoclassical economists have responded to this critique by:

- Encouraging ignorance of the economics literature, the history of thought, and scholarship in general so as to ignore the critique.
- Misrepresenting the critique as addressing only the measurement problem of constructing an adequate index number for capital.
- Discarding long run Neoclassical theories, aggregate macroeconomic theories, and the marginal productivity theory of distribution in favor of disaggregated intertemporal and temporary General Equilibrium models, in which nearly anything can happen and nothing need happen.

The Cambridge Capital Controversy is generally agreed to have exposed certain problems with Neoclassical capital theory. For example, [Bohm-Bawerk's](#) theory in which the interest rate is determined by the interaction of subjective time preferences and the supposedly greater productivity of techniques with a longer period of production was shown to be incoherent in multicommodity models. More recent capital theories shown to be incoherent include [Robert Solow's growth theory](#) and his formulation of [Irving Fisher's](#) capital theory in which interest rates are the price of intertemporal trades.

Neo-Ricardians have yet to formulate a definitive critique of short-run disaggregated theories. [Garegnani](#) has recently suggested that a sequence of temporary equilibria can exhibit behavior Neoclassical economists find perverse. [Bertram Schefold](#) argues that the Cambridge Capital Controversy is related to stability problems in neoclassical models of intertemporal equilibrium paths. A general conclusion of the critique is that equilibrium prices are not scarcity indices. A higher price of an input into production is not necessarily associated with a tendency for firms to adopt techniques that use that input less intensively. Nor need firms necessarily increase the production of commodities that use that input less intensively and decrease the production of commodities that use that input more intensively.

6.0 What is the Transformation Problem?

The transformation problem is how to transform relationships in the system of labor values into relationships in the system of prices of production. The principle relationships to have been investigated in the literature revolve around:

- Relative prices and values
- The rate of profit
- Total surplus value and total profits
- Total value and total prices.

All of these quantities have been investigated in input/output models that abstract from concrete phenomena characteristic of actually existing capitalism, such as monopolies. Some Marxists have questioned whether quantitative relationships should be expected to hold between values and prices since they belong to different levels of analysis.

6.1 Why would one expect prices of production to differ from labor values?

Because of the variability of [organic composition of capital](#) among different industries:

Suppose I employ twenty men at an expense of 1000 pounds for a year in the production of a commodity, and at the end of the year I employ twenty men again for another year, at a further expense of 1000 pounds in finishing or perfecting the same commodity, and that I bring it to market at the end of two years, if profits be 10 per cent., my commodity must sell for 2,310 pounds.; for I have employed 1000 pounds capital for one year, and 2,100 pounds capital for one year more. Another man employs precisely the same quantity of labour, but he employs it all in the first year; he employs forty men at an expense of 2000 pounds, and at the end of the first year he sells it with 10 per cent. profit, or for 2,200 pounds. Here then are two commodities having precisely the same quantity of labour bestowed on them, one of which sells for 2,310 pounds - the other for 2,200 pounds. ([David Ricardo 1821, Chapter I, Section IV](#))

So if market prices tend towards prices of production, market prices will not tend towards relative labor values. Marx thought Ricardo did not totally understand his own examples on this point. According to Marx, Ricardo mistakenly emphasized how relative prices alter with variations in wages, rather than the differences between relative prices of production and relative labor values at any positive rate of profits:

What [Ricardo] does in fact examine is this: supposing that cost-prices *differ* from the values of commodities - and the assumption of a *general rate of profit* presupposes this difference - how in turn are these cost-prices (which are now, for a change, called "relative values") themselves reciprocally modified by the rise or fall in wages, taking also into account the varying proportions of the organic component parts of capital? If Ricardo had gone into this more deeply, he would have found that

- owing to the diversity in the organic composition of capital which first manifests itself in the immediate production process as the difference between variable and constant capital and is later enlarged by differences arising from the circulation process - the mere existence of a *general rate of profit* necessitates *cost-prices* that differ from *values*. He would have found that, even if *wages* are assumed to *remain constant*, the difference exists and therefore is *quite independent* of the rise or fall in wages. ([Karl Marx 1968, Chapter X, Section A.4.a](#))

Anyways, Marx understood that the LTV, considered as a determination of relative prices, cannot be expected to be true. But Ricardo and Marx investigated the LTV for reasons connected with distribution, accumulation, laws of motion of capitalist economies, and an explanation of the source of profits.

6.2 When are prices of production equal to labor values?

When the rate of profits is zero or when the [organic composition of capital](#) does not vary between industries.

6.3 What was Marx's algebraic solution to the transformation problem?

Marx's solution to the transformation problem is in chapter 9 of the [third volume](#) of *Capital*, in [Part II \(Chapter VIII, Section 6 and Chapter XV, Section 5\)](#) of *Theories of Surplus Value*, and in a [letter to Engels \(2 August 1862\)](#). Marx presents his solution by means of various examples, the most straightforward from the third volume being presented in the table below.

Example 6.1 Illustrates Marx's Solution					
Sphere of Production	Capitals	Surplus Value	Labor Value	Price of Production	Divergence
I.	$70c + 30v$	30s	130	120	-10
II.	$80c + 20v$	20s	120	120	0
III.	$90c + 10v$	10s	110	120	+10
Total	$240c + 60v$	60s	360	360	0

The economy is assumed to be composed of three sectors of varying [organic compositions of capital](#) (second column). Note that the second sector has an average organic composition of capital. The quantities of surplus value shown in the third column are derived under the assumption that the [rate of surplus value](#) is 100%. The ratio of total surplus value to the total value of capital is 20%. Marx [mistakenly](#) derives the rate of profits from a system with constant capital costed up with embodied labor values, thereby assuming the rate of profits is 20%. Since the value of capital is 100 in each sector, Marx sets the price of production of the output of each sector to 120.

As can be seen in the table, prices of production deviate from labor values except for a sector with an average [organic composition of capital](#). The labor value of total output,

though, is equal to total output when evaluated in prices of production, 360. Likewise, under Marx's procedure, total profits are equal to total surplus value, 60. If Marx were correct, surplus value would be generated from exploitation of the worker and then redistributed in the form of profits. This redistribution would result in prices of production that deviate from labor values so as to obtain equal rates of profits in all sectors. In short,

[the] average rate of profit can and must come about, not only without violating the law of value, but precisely on the basis of this law... (From Frederick Engels' preface to [Volume 2](#) of *Capital*)

6.4 Why was Marx's solution to the transformation problem inadequate?

Marx calculated the rate of profit on inputs evaluated at untransformed labor values. Marx seems to accept that his arithmetic examples are not fully worked out:

The development given above also involves a modification in the determination of a commodity's cost price. It was originally assumed that the cost price of a commodity equalled the *value* of the commodities consumed in its production. But for the buyer of a commodity, it is the price of production that constitutes its cost price and can thus enter into forming the price of another commodity. As the price of production of a commodity can diverge from its value, so the cost price of a commodity, in which the price of production of other commodities is involved, can also stand above or below the portion of its total value that is formed by the value of the means of production going into it. It is necessary to bear in mind this modified significance of the cost price, and therefore to bear in mind too that if the cost price of a commodity is equated with the value of the means of production used up in producing it, it is always possible to go wrong. ([Karl Marx 1894, Chapter 9](#))

When the inputs are evaluated at their prices of production, equalities that Marx relied on no longer obtain in the dual system approach. If the same prices of production are used for the inputs to and the outputs from production processes, the correct rate of profit will generally not be equal to Marx's rate of profit in the value schema ($S/(C + V)$). Total profits will generally deviate from total surplus value. And the total labor value of the national product will deviate from its price when all commodities are evaluated at prices of production. (One of these latter two equalities can be imposed by choosing an appropriate [numeraire](#).)

6.5 Can you provide an example in which the LTV is valid?

Consider a simple capitalist economy in which wheat and iron are produced by the production processes described in the following table. Notice that if both processes are operated at an unit level, twice as many workers are employed in the iron industry as the wheat industry. When the processes are operated in the ratios shown, the net output consists solely of 204 qr. wheat. Since this output is produced by 272 workers, the labor embodied in each quarter wheat is $1 \frac{1}{3}$ person years.

Example 6.2 Quantity Flows						
30 qr. wheat	&	15 t. iron	&	240 workers	->	240 qr. wheat
6 qr. wheat	&	1 t. iron	&	32 workers	->	16 t. iron

The following table shows a vertically integrated iron industry for the same processes operated at different ratios. 102 workers produce a net output of 272 t. iron. So the labor value of each ton of iron is $2\frac{2}{3}$ person-years. Note that the ratio of the labor value of iron to wheat is the same as the ratio of direct labor inputs. This equality always holds in these circulating capital models when the organic composition of capital is constant across industries.

Example 6.2 Quantity Flows for Iron Subsystem						
6 qr. wheat	&	3 t. iron	&	48 workers	->	48 qr. wheat
42 qr. wheat	&	7 t. iron	&	224 workers	->	112 t. iron

Prices of production satisfy the following system of equations:

$$(30pw + 15pi + 240w)(1+r) = 240pw$$

$$(6pw + 1pi + 32w)(1+r) = 16pi$$

Given a certain choice of numeraire, the solution is as follows:

$$pw = \$1.33$$

$$pi = \$2.67$$

$$w = (3-r)/[3(1+r)] \text{ or } r = 3(1-w)/(1+3w)$$

Notice that prices of production for this example do not vary with the distribution of income. Furthermore, prices of production are equal to labor values in this example. Having calculated labor values and prices of production, one can construct the following table for this example:

Comparison of Value and Price Accounting for Example 6.2				
Sector	Constant Capital	Variable Capital	Surplus Value/Profit	Value/Price of Output
Wheat	80	240 w	240 (1 - w)	320
Iron	$10\frac{2}{3}$	32 w	32 (1 - w)	$42\frac{2}{3}$
Total	$90\frac{2}{3}$	272 w	272 (1 - w)	$362\frac{2}{3}$

It does not matter in the above table whether one calculates in labor values or prices of production; the entries are the same. Notice that the ratio of the value of variable capital to constant capital is the same in both industries. This observation is another manifestation of the constancy of the organic composition of capital across sectors.

Finally, one can express the wage as a function of how much labor time is obtained free by the capitalists:

$$w = 1/(1 + e)$$

where e is the rate of exploitation. When the workers obtain pay for all time worked, the rate of exploitation is zero and the wage is unity. If physical quantity flows remain unchanged, a lower wage results from the capitalists paying for less of the time that the labor force works.

6.6 Can you provide an example in which prices of production are not proportional to labor values?

The same example was used to illustrate the calculation of labor values and prices of production. Since the organic composition of capital differs between sectors in this example, it provides an illustration of difficulties with the LTV. The following table shows the quantity flows in this example:

Example 6.3 Quantity Flows						
74 qr. wheat	&	37 t. iron	&	592 workers	->	592 qr. wheat
18 qr. wheat	&	3 t. iron	&	48 workers	->	48 t. iron

Comparison of Value and Price Accounting. It was shown above that the labor embodied in wheat is $1 \frac{13}{51}$ person-years per quarter and that the labor embodied in iron is $1 \frac{29}{51}$ person-years per ton. Assume that the value of the yearly wage is equal to the proportion of the net output it can purchase. Then one can calculate the labor value quantities shown in the following table:

Value Accounting for Example 6.3				
Sector	Constant Capital	Variable Capital	Surplus Value	Value of Output
Wheat	$150 \frac{46}{51}$	592 w	$592 (1 - w)$	$742 \frac{46}{51}$
Iron	$27 \frac{5}{17}$	48 w	$48 (1 - w)$	$75 \frac{5}{17}$
Total	$178 \frac{10}{51}$	640 w	$640 (1 - w)$	$818 \frac{10}{51}$

Prices of production were also calculated above. Notice that prices of production are equal to labor values only when the rate of profits are zero. The following table is constructed using the physical quantity flows and prices of production:

Price Accounting for Example 6.3				
Sector	Price of Constant Capital	Wages	Profits	Price of Output
Wheat	$2,960 (13 + r)/(255 + r)$	592 w	$2,960 (51 - r)/(255 + r) - 592 w$	$189,440/(255 + r)$
Iron	$240 (29 + r)/(255 + r)$	48 w	$240 (51 + 15 r)/(255 + r) - 48 w$	$3,840 (5 + r)/(255 + r)$
Total	$640 (71 + 5 r)/(255 + r)$	640 w	$640 (1 - w)$	$1,280 (163 + 3 r)/(255 + r)$

Only one of Marx's equalities is satisfied for this data. Total surplus value is equal to total profits. This equality was established by choosing the net output per worker as the numeraire. At wages corresponding to positive profits, the prices of outputs and capital goods differs from labor values. Consequently, the total labor value of output differs from the total price of output for this example. Also, the rate of profit is not $S/(C + V)$. Does this

re-evaluation of output invalidate Marx's theory that the source of profits is the exploitation of the worker? Notice that profits are positive if and only if the rate of exploitation is positive.

More on the Organic Composition of Capital. Prices of production are proportional to labor values when the [organic composition of capital](#) is [constant](#) across industries. When this condition is met, capital per worker is also constant across industries, where capital per worker is calculated by evaluating per capita capital goods at prices of production. Since relative prices deviate from labor values in this example, capital per worker must also vary across industries.

The example illustrates this variation. Using the data in the price accounting table and the known relationship between wages and the rate of profits, the capital intensity for this economy as a whole, k , is established to be

$$k = (71 + 5r) (1 + r) / [5 (3 - r) (17 + r)].$$

The wheat industry is less capital-intensive than average, and the iron industry is more capital-intensive. The difference between capital-intensities in these industries and the average capital intensity is (in obvious notation):

$$k - k_w = 6 (1 + r) / [5 (3 - r) (17 + r)]$$

$$k_i - k = 74 (1 + r) / [5 (3 - r) (17 + r)]$$

It is interesting to compare these deviations of the wheat and iron industries from the average capital intensity with the deviations of the vertically integrated wheat and iron industries. It was shown [above](#) how to reapportion the given quantity flows so as to create two subsystems whose net outputs are wheat and iron, respectively. Wages and the prices of the capital goods can be totalled in each subsystem to obtain capital per worker for each subsystem. The vertically integrated wheat industry is still less capital intensive than average, and the vertically integrated iron industry remains more capital intensive. The capital intensities of the vertically integrated wheat industry and the vertically integrated iron industry differ from the average capital intensity, respectively, by

$$(1 + r) / [5 (3 - r) (17 + r)]$$

and

$$50 (1 + r) / [5 (3 - r) (17 + r)]$$

So the capital intensity of the wheat industry differs from the average six times as much as the capital intensity of the vertically integrated wheat industry. The capital intensity of the iron industry differs only 1.48 times as much as the difference of the vertically integrated iron industry and the average capital intensity. The analytical process of vertical integration seems to reduce variations among industries in the value of capital per head.

The point of these observations is difficult to explain without advanced mathematics. The LTV is a valid theory of price when both direct labor coefficients and labor values are

eigenvectors of the input-output matrix corresponding to the [Perron-Frobenius](#) root of the input-output matrix. If the difference between the average capital intensity and the capital intensity of vertically integrated industries is usually smaller than the difference between the average capital intensity and the capital intensity of non-vertically integrated industries, then labor values will usually be a better approximation than direct labor inputs to an eigenvector for the Perron-Frobenius root of the input-output matrix. That is, the LTV will usually be a good approximate theory of price.

On the Production of a Commodity of Average Organic Composition. One can also decompose the given quantity flows to consider the production of a commodity of average organic composition, in some technical sense different from Marx's. The other sector produces the wheat remaining in the gross output. The following table shows the production of a composite commodity of average organic composition.

Example 6.3 Modified Quantity Flows				
18 qr. wheat	&	9 t. iron	&	144 workers -> 144 qr. wheat
18 qr. wheat	&	3 t. iron	&	48 workers -> 48 t. iron

Total capital in this sector consists of 36 qrs. wheat and 12 t. iron, a ratio of three to one. Gross and net outputs are also in this ratio. This is a defining property of [Piero Sraffa's](#) "standard commodity." Since these quantity flows result from a mere rescaling of the given production processes, the labor values of wheat and iron remain unchanged. Labor values can be used to evaluate inputs and outputs, resulting in the following table:

Value Accounting for Modified Example 6.3				
Sector	Constant Capital	Variable Capital	Surplus Value	Value of Output
Wheat	36 12/17	144 w	144 (1 - w)	180 12/17
Iron	27 5/17	48 w	48 (1 - w)	75 5/17
Total	64	192 w	192 (1 - w)	256

Prices of production need to be recalculated for a new numeraire, that being the net output per worker of the standard system. Prices of production are then found to be:

$$\begin{aligned}
 p_w &= 64 / [3 (17 + r)] = 16 (3 w + 1) / [3 (12 w + 5)] \\
 p_i &= 16 (5 + r) / [3 (17 + r)] = 16 (3 w + 2) / [3 (12 w + 5)] \\
 w &= (3 - r) / [3 (1 + r)] \\
 r &= 3 (1 - w) / (3 w + 1)
 \end{aligned}$$

When the rate of profits is zero, the wage is unity. Notice that prices of production are equal to labor values only if the rate of profits is zero. Evaluating inputs and outputs with prices of production yields the following table:

Price Accounting for Modified Example 6.3				
Sector	Price of Constant Capital	Wages	Profits	Price of Output
Wheat	$48(9w + 4)/(12w + 5)$	144 w	$576(3w + 1)(1 - w)/(12w + 5)$	$768(3w + 1)/(12w + 5)$
Iron	$16(21w + 8)/(12w + 5)$	48 w	$192(3w + 2)(1 - w)/(12w + 5)$	$256(3w + 2)/(12w + 5)$
Total	64	192 w	$192(1 - w)$	256

All Marx's equalities hold for this modification of the example. Total value equals the total price of the standard commodity. Total surplus value equals total profits. Individual price-value deviations result in the prices of output and capital goods varying from labor values in individual sectors. But these differences cancel out for the economy as a whole when the economy is in standard proportions. One can calculate the rate of profit indifferently in labor values, physical quantities, or prices of production.

Of course, one cannot expect an economy to be in "standard proportions." Can one still say that the total value added in any period is the labor expended in that period, due allowance being taken for [variations in skill](#)? The prices of capital goods will differ from their labor values in individual industries. Since the economy is not in standard proportions, the total labor value of capital will not be its price. Nor will the total labor value of output be equal to its price. But the price of outputs in each industry and in the economy as a whole will be the sum of the price of capital goods and labor inputs. Furthermore, although labor contributes total value-added, the workers are not paid the entire net output. Perhaps these observations are enough to justify Marx's theory of value.

The New Interpretation Applied to This Example. Marx's equalities can also be justified by the "[New Interpretation](#)." Since the net output per worker was chosen as the numeraire when calculating [prices of production](#), a dollars worth of net output represents, in some sense, one person year. In the technical terminology of the New Interpretation, the *Monetary Expression of Value* is unity. Notice that the Monetary Expression of Value can be calculated with any set of prices; it need not be found only for prices of production.

The labor value of constant capital was previously calculated as the labor embodied in the means of production. According to the New Interpretation, the labor value of constant capital should rather be calculated as the labor value expressed by the money with which the capitalists purchase the means of production. Likewise, the labor value of variable capital should be calculated as the labor expressed by the money with which the capitalists purchase labor-power, not the labor embodied in the commodities the workers buy with their wages. (The example has already used this method of calculating the labor value of variable capital.) The following table shows labor values calculated under the guidance of the New Interpretation.

Value Accounting for Example 6.3 Under the New Interpretation				
Sector	Constant Capital	Variable Capital	Surplus Value	Value of Output
Wheat	$2,960(13 + r)/(255 + r)$	592 w	$592(1 - w)$	$1,184(160 + 3r)/(255 + r)$
Iron	$240(29 + r)/(255 + r)$	48 w	$48(1 - w)$	$96(200 + 3r)/(255 + r)$
Total	$640(71 + 5r)/(255 + r)$	640 w	$640(1 - w)$	$1,280(163 + 3r)/(255 + r)$

The New Interpretation does not provide an alternative approach to calculating prices of production; they are calculated as [above](#). Labor values of individual commodities are found as the sum of the labor value of constant capital, the labor value of variable capital, and surplus value. Since surplus value is generated in proportion to direct labor inputs, the labor values of individual commodities differ from their prices of production. Yet total labor value equals total prices of production, and total surplus value equals total profit. Furthermore, the rate of profit in the terms of labor values is numerically equal to the rate of profit used in calculating prices of production. All of Marx's equalities are vindicated by this controversial interpretation.

The [Temporal Single System](#) approach offers another way of thinking about the arithmetic of the transformation problem. When Marx sets prices equal to values in Volume 1 of *Capital*, he determines both labor values and prices as the labor embodied in commodities. The transformation in Volume 3 is not a transformation merely of labor values to prices of production. Rather, both labor values and prices are transformed to new quantities in a single system in which capitalists obtain profit in each industry in proportion to the capital advanced. Although contravening a century's interpretation of Marx, proponents of the Temporal Single System approach argue with style for their reading of Marx.

6.7 What empirical evidence supports the LTV?

As noted [above](#), the LTV may be a good approximate theory of prices if the price of capital per worker for vertically integrated industries is closer to the average price of capital per worker than for non-vertically integrated industries. [Anwar Shaikh](#) has checked this condition with the 1947 input-output table collected by [Wassily Leontief](#) for the United States. This input-output table divides the U.S. economy into 190 sectors. Anwar Shaikh found that the ratio of the standard deviation of the capital-labor ratios to the mean capital-labor ratio (the coefficient of variation) is 1.14. The coefficient of variation for capital-labor ratios of vertically integrated sectors is 0.60. So the desired condition is confirmed. Furthermore, 96% of sectoral variations in the logarithm of relative prices are explained by variations in the logarithm of relative labor values. (Anwar Shaikh provided a theoretical argument for using natural logarithms.) Anwar Shaikh also tested the ability of the LTV to explain temporal variations in prices. He used Italian input-output tables developed by Graziella Marzi and Paolo Varri for 1959 and 1969. These tables contain 25 sectors. Anwar Shaikh found that 92% of the temporal variation of the logarithm of prices of production is explained by variations in the logarithm of relative labor values.

[Paul Cockshott](#), [Allin Cottrel](#), [Ed Ochoa](#), and [Pavle Petrovic](#) are some other economists whose recent empirical results support the LTV. [Edward Wolff](#) found empirical evidence against Marx's law of the tendency of the rate of profits to fall. He argues, though, that Marx's theory provides an adequate framework for empirical exploration of such issues. [Anwar Shaikh and E. Ahmet Tonak](#) found empirical evidence in favour of Marx's law. In other empirical work, [William Nordhaus](#) used a Smithian labor-commanded standard to measure technological change. [Ian Steedman and Judith Tomkins](#) found empirical evidence against the LTV in their sophisticated formulation of the question.

7.0 What can I Read to Find Out More About the LTV?

The LTV, as presented in this FAQ, is a mostly 20th century, mostly Anglo-American interpretation of Classical economics and [Marx](#). The most important Classical economists, in this context, are [Adam Smith](#) and [David Ricardo](#). [Eugen von Bohm-Bawerk](#) produced one of the first and most influential critiques of Marx from a neoclassical perspective. [Paul Sweezy](#) made these criticisms available to an English-speaking audience in 1949. From the perspective of this FAQ, Sweezy's decision to include an essay by [Ladislaus von Bortkiewicz](#) was even more important. [Von Bortkiewicz](#) laid out the mathematical framework of the transformation problem in terms of dual systems of labor values and prices of production. There were other similar early criticisms of Marx, for example, by [V. K. Dmitriev](#), and even another [more general essay](#) by von Bortkiewicz that was not restricted to the specific sectors examined in the essay Sweezy chose to translate.⁴ Awareness, however, of this other non-English language literature only came later among Anglo-American economists.

[J. Winternitz](#) developed one response to Sweezy's presentation of the problem. He generalized von Bortkiewicz's statement of the problem, and showed that restrictions imposed by Marx's Volume 2 reproduction schemes are irrelevant to the transformation problem. Winternitz and, in response, [Kenneth May](#) unnecessarily restricted themselves to three-sector economies. A 1957 essay by [Francis Seton](#) further developed the standard dual-system interpretation. Seton presented a general N-commodity model. He showed that in the dual system setting, all of Marx's "normalization" properties generally do not hold simultaneously. For example, either total surplus value equals total profits, or total values equal total output evaluated in prices of production, but not both. A book by [Ronald Meek](#) is a good example of an English defense of Marx of that era.

One of the greatest 20th century mathematicians, [Johnny Von Neumann](#), comes into the story here. He presented a growth model that some economists think is a formalization of Classical theory. Despite the misleading translation of the title, the English-language translation may be superior to the German original in that it is accompanied by David Champernowne's commentary. The Classical contextualization of the Von Neumann growth model is heavily influenced by [Piero Sraffa's](#) independently-developed 1960 masterpiece. Prices of production are determined in these models, given physical quantity flows and the wage; labor values are not mentioned. [Nobuo Okishio](#) provided another seminal paper of that era. Okishio's paper, which I have not read, "proved" that Marx's theory of the falling rate of profit is not logically valid. Okishio's paper is in Japanese, and Japan has long had an established tradition of Marxist economics.

Critiques of Marx produced in the 1970s brought together some strands of this literature. [Paul Samuelson](#) presented one such critique from what he termed a Sraffian position. (See also Samuelson's debate with [William J. Baumol](#) and [Michio Morishima](#).) In an angry bit of mathematics, [Ian Steedman](#) added the charge of redundancy to the traditional assertion of inconsistency of Marx's theory. Steedman argued not only that Marx's normalization

⁴ Ladislaus Von Borkiewicz. Wertrechnung und Preisrechnung im Marxschen System [Value and Price in the Marxian System]. *Archiv für Sozialwissenschaft und Sozialpolitik*; vol. XXV, 1907, pp. 10-51; 445-488.

conditions are inconsistent in the system of prices of production, but that a materialist theory could be developed using prices of production and no reference to labor values.

Three important recent schools of thought reacted to these findings. [G. A. Cohen](#), [Jon Elster](#), and [John Roemer](#) are the most prominent developers of "[Analytical Marxism](#)." Analytical Marxists use the techniques of mainstream economics, particularly General Equilibrium theory, to evaluate Marx. Analytical Marxists provide criticisms of many points of detail, but Jon Elster, at least, seems to find promising Marx's accounts of technical change, exploitation, and the sociology of knowledge. [Duncan Foley's](#) and Gerard Dumenil's "[New Interpretation](#)" of the transformation problem makes it seem much less problematic. The New Interpretation emphasizes the role of money in Marx's theory of value. Under this interpretation, all of Marx's normalization conditions hold. [Guglielmo Carchedi](#), [Alan Freeman](#), [Andrew Kliman](#) and other developers of the Temporal Single System (TSS) approach also present an interpretation in which Marx's normalization conditions are logically consistent. The TSS emphasizes the use of dynamic models in interpreting Marx.

At the end of the day, where are we? [Neoclassical value theory](#) is incoherent. [Prices of production](#) are internally consistent and may be used for a materialist theory of value. It is not clear how important labor values are to such a theory, or whether it can properly be described as a labor theory of value. Marx's legacy is a matter of contemporary lively debate among some economists. Perhaps other elements of a theory of value are to be found in Chapter 17 of Keynes' [General Theory](#) or in (Old) [Institutionalism](#). Or perhaps the questions addressed by theories of value should be [reformulated](#).

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