Calculation in-Natura, from Neurath to Kantorovich

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1 Introduction

Following the collapse of hither too existing socialism in Eastern Europe and Central Asia. There was a crisis in socialist economic thought. If we contrast the situation of the 1990s with what had existed 40 years earlier, we see that whilst in the 1950s, socialism and economic planning were almost universally accepted, even by enemies of socialism, as being viable ways to organize an economy, by the 1990s the reverse applied. Among orthodox opinion it was now taken for granted that socialism was the 'god that failed', and that socialist economic forms, when judged in the balance of history had been found wanting. And among socialist theorists there was a general retreat from ideas that had previously been taken for granted, a movement towards market socialist ideas, an accommodation with the idea that the market was a neutral economic mechanism.

Whilst accommodation to the market was, to anyone familiar with Marx, completely at odds with his critique of civil society[44], it nonetheless gained considerable credence. Former governing socialist parties, thrown suddenly into opposition in renascent capitalist states, felt that they had to restrict their ambitions to reforms within a market economy.

In retrospect one can see that the mid 1970s represented the high water mark of the socialist tide. Whilst the Vietnamese were driving the US out of Saigon, and the last colonial empire in Africa, that of Portugal, was falling, the collapse of the cultural revolution in China was setting the economic scene for the triumph of capitalism in the 80s and 90s. When, after the death of Mao, Deng threw open the Chinese economy to western capital investment, the balance of economic forces across the whole world was upset. An immense reserve army of labour, hireable of the lowest of wages, was thrown onto the scales. The bargaining position of business in its struggles with domestic labour movements was, in one country after another, immensely strengthened.

The general intellectual/ideological environment today is thus much less favorable to socialism than it was in the 20th century. This is not merely a consequence of the counter-revolutions that occurred at the end of the 20th century, but stems from a new and more vigorous assertion of the classic tenets of bourgeois political economy. This re-assertion of bourgeois political economy not only transformed economic policy in the West, but also prepared the ideological ground for counter revolutions in the East.

The theoretical preparation for the turn to the free market that occurred in the 1980s had been laid much earlier by right wing economic theorists like Hayek and Friedman. Their ideas, seen as extreme during the 1950s and 60s gained influence through the proselytizing activities of organizations like the Institute for Economic Affairs and the Adam Smith Institute. These groups produced a series of books and reports advocating free market solutions to contemporary economic problems. They won the ear of prominent politicians like Margaret Thatcher, and from the 1980s were put into practice. She was given the liberty to do this by a combination of long term demographic changes and short term conjectural events. Within Britain, labour was in short supply, but across Asia it had become super abundant. Were capital free to move abroad to this plentiful supply of labour then the terms of the exchange between labour and capital in the UK would be transformed. Labour would no longer hold the stronger bargaining position. The conjunctural factor making this possible was the surplus in foreign trade generated by North Sea oil. Hitherto, the workers who produced manufactured exports had been essential to national economic survival. With the money from the North Sea, the manufacturing sector could be allowed to collapse without the fear of a balance of payments crisis.

The deliberate run-down of manufacturing industry shrank the social basis of social democracy and weakened the voice of labour both economically and politically.

The success of Thatcher in attacking the trades union movement in Britain encouraged middle class aspiring politicians in the East like Vaclav Klaus and presaged a situation in which Hayekian economic doctrines would become the orthodoxy. Thatcher's doctrine TINA, There Is No Alternative, (to capitalism) was generally accepted.

The theoretical dominance of free market economic ideas had by the start of the 21st century become so strong, that they were as much accepted by social democrats and self professed communists, as they had been by Thatcher. They owe dominance both to class interests and to their internal coherence. The capitalist historical project took as its founding documents the Declaration of the Rights of Man, and Adam Smith's Wealth of Nations. Together these provided a coherent view of the future of Bourgeois or Civil Society, as a self regulating system of free agents operating in the furtherance of their private interests. Two centuries later when faced with the challenge of communism and social democracy, the more farsighted representatives of the bourgeoisie returned to their roots, restated the original Capitalist Manifesto, and applied it to current conditions. The labour movement by contrast had no such coherent social narrative. Keynes's economics had addressed only technical issues of government monetary and tax policy, it did not aspire to the moral and philosophical coherence of Smith.

The external economic and demographic factors that originally favored the turn to the market are gradually weakening. Within the next 20 years the vast labour reserves of China will have been largely utilized, absorbed into capitalist commodity production. Globally we are returning to the situation that Western Europe had reached a century ago: a maturing world capitalist economy in which labour is still highly exploited but is beginning to become a scarce resource. These were the conditions that built the social cohesion of classical social democracy, the conditions that gave rise to the IWW and then CIO in America, and led to the strength of communist parties in Western Europe countries like France, Italy and Greece post 1945. We see perhaps, in South America, this process in operation today.

These circumstances set 21st century critical political economy a new historical project: to counter and critique the theories of market liberalism as effectively as Marx critiqued the capitalist economists of his day.

The historical project of the world's poor can only succeed if it promulgates its own political economy, its own theory of the future of society. This new political economy must be as morally coherent as that of Smith, must lead to economically coherent policy proposals, which if enacted, open the way to a new post-capitalist civilisation. As those of Smith opened the way to the post feudal civilisation.

Critical political economy can no longer push to one side the details of how the non-market economy of the future is to be organised. In the 19th century this was permissible, not now. We can not pretend that the 20th century never happened, or that it taught us nothing about socialism. In this task 20th century Western critical Marxists like Cliff, Bettleheim or Bordiga will only take us so far. Whilst they could point out weaknesses of hitherto existing socialism, they did this by comparing it to an ideal standard of what these writers thought that a socialist society should achieve. In retrospect we see that these trends of thought were a product of the special circumstances of the cold war, a striving for a position of ideological autonomy 'neither Moscow nor Washington', rather than a real contribution to political economy. The very psychological detachment that such writers sought, deflecting from their own heads the calumnies directed at the USSR, prevented them from positively engaging with the problems faced by historically existing socialism. It is only if you envisage being faced with such problems oneself, that one would come up with practical answers:

> "It is not the critic who counts: not the man who points out how the strong man stumbles or where the doer of deeds could have done better. The credit belongs to the man who is actually in the arena, whose face is marred by dust and sweat and blood, who strives valiantly, who errs and comes up short again and again, because there is no effort without error or shortcoming, but who knows the great enthusiasms, the great devotions, who spends himself for a worthy cause; who, at the best, knows, in the end, the triumph of high achievement, and who, at the worst, if he

fails, at least he fails while daring greatly, so that his place shall never be with those cold and timid souls who knew neither victory nor defeat." (Citizenship in a Republic, Roosevelt)

In the 19th century Marx's Capital was a critique of the political economy that underlay British Liberalism. 21st century critical political economy must perform an analogous critique of neo-liberal economics comparable in rigour and moral depth. In particular it must engage with the ideas of the Austrian school: Boehm-Bawerk, Mises, Hayek, whose ideas now constitute the keystone of conservatism. Soviet Marxism felt strong enough to ignore them then, and the response in the West came in the main from marginalist socialists like Lange and Dickinson. If socialism is to reconstitute itself as the commonsense of the 21st century - as it was the commonsense of the mid 20th, then these are the ideas that must be faced[1].

In attacking them one should not hesitate to use the advances in other sciences - statistical mechanics, information theory, computability theory. And, to re-establish *scientific* socialism there must be a definitive break with the speculative philosophical method of much of Western Marxism. From the time of Marx till about the mid-twentieth century, most left intellectuals saw socialism and science as going hand-in-hand, in some sense. Most scientists were not socialists (though some prominent ones were), but Marxists seemed to regard science as friendly to, or consonant with, their project, and even saw it as their duty as materialists to keep current with scientific thought and assess its implications for social questions.

But since some point in the 1960s or thereabouts, many if not most Western Marxist thinkers have maintained a skeptical or hostile attitude towards science, and have drawn by preference on (old) philosophical traditions, including Hegelianism. It is not clear why this has occurred but these may be some of the factors:

- The conception of science as socially embedded. Science in bourgeois society is bourgeois science, rather than offering privileged access to an independent reality. This idea was obviously present in the Proletlult tendency criticised by Lenin, and was later expressed in Lysenkoism. In addition there has been a conflation of science and technology in the minds of many writers. The role of nuclear weapons no doubt played a part in this and spilled over to a general hostility to nuclear power. Socio-biology too, was seen as hostile to progressive social thought, so the alliance between Marxism and Darwinism came to be weakened. Evolutionary psychology could be seen as transparent apologetics (for example [47]), but this blinded left thinkers to progressive Darwinists like Dawkins[15, 14].

- Althusser, the French communist philosopher was obviously pro-scientific in intent, but may have unwittingly influenced many of his followers in

[1]This article is part of a systematic program of work aimed at contributing to this critique, previous articles were [10, 5].

a contrary direction. One could easily get the impression from Althusser that while staying too close to Hegel is an error, empiricism is a cardinal sin. Equate empiricism and science, and you're off to the races.

- The appropriation of the "Scientific Socialism" label by the USSR and its official ideologists.

- The brute historical fact that while science was doing very nicely, socialism in the West was not. Thus undermining the idea that Marxism and science somehow marched together.

Whatever exactly is the cause, the effect is that while in the 1930s (say) one might have expected the "typical" young Marxist intellectual to have a scientific training – or at least to have general respect for scientific method – by the turn of the century one would be hard pressed to find a young Marxist intellectual (in the dominant Western countries) whose background was not in sociology, accountancy, continental philosophy, or perhaps some "soft" (quasi-philosophical) form of economics, and who was not profoundly skeptical of (while also ignorant of) current science[2].

Unlike that Western Marxist tradition have to treat political economy and the theory of social revolution like any other science. We must formulate testable hypotheses, which we then asses against empirical data. Where the empirical results differ from what we expected, we must modify and retest our theories[3].

In addition we must recover and celebrate the advances in political economy that arose from the Russian experience: the method of material balances used in preparing the 5 year plans and systematized as Input Output analysis by Leontief; the method of linear programming pioneered by Kantorovich; the time diaries of Strumlin.

In this article I focus particularly on recovering the work of Kantorovich the only Soviet Economic Nobelist, and showing that his work provided a fundamental theoretical response to von Mises. Kantorovich was an eminent mathematician, whose work went well beyond economics, but in this article I focus only on his economic contributions. In explaining these I reproduce in section 3.3 some of his original numerical examples drawn from his experience in Soviet heavy industry. I have avoided, however, giving any detailed presentation of the mathematical techniques (algorithms) that Kantorovich and Dantzig developed, both because I assume that the readership are not specialists in linear algebra, and secondly because these techniques have now been packaged up in open-source software that can be used by non-algebraists. I give in sections 3.3 and 3.4 what is essentially a tutorial introduction to using such package to solve planning problems. I summarise what these mathematical techniques mean in practical terms. What type of economic problem do they allow us to solve?

[2]I owe the above argument about Western Marxism to my co-worker Allin Cottrell.
[3]For work in the vein see [6, 38, 7, 63, 64, 51, 8].

As illustrations I will focus on how Kantorovich allows us to pose problems of national or continental environmental trade-offs. From this I go on to ask how do his ideas relate to the Austrian critique of socialism?

What are their implications for the future of economic planning?

How has the field advanced since Kantorovich's day, and what are the *political* implications of these advances?

2 What is economic calculation?

In contemporary society the answer seems simple enough: economic calculation involves adding up costs in terms of money. By comparing money costs with money benefits one may arrive at a rational - wealth maximizing - course of action.

In a famous paper[58] the Austrian economist Mises argued that it was only in a market economy in which money and money prices existed, that this sort of economic rationality was possible.

His claims were striking, and, if they could be sustained, apparently devastating to the cause of socialism. The dominant Marxian conception of socialism involved the abolition of private property in the means of production and the abolition of money, but Mises argued that "every step that takes us away from private ownership of the means of production and the use of money also takes us away from rational economics" ([58]: 104). The planned economy of Marx and Engels would inevitably find itself "groping in the dark", producing "the absurd output of a senseless apparatus" (106). Marxists had counterposed rational planning to the alleged 'anarchy' of the market, but according to Mises such claims were wholly baseless; rather, the abolition of market relations would destroy the only adequate basis for economic calculation, namely market prices. However well-meaning the socialist planners might be, they would simply lack any basis for taking sensible economic decisions: socialism was nothing other than the "abolition of rational economy".

As regards the nature of economic rationality, it is clear that Mises has in mind the problem of producing the maximum possible useful effect (satisfaction of wants) on the basis of a given set of economic resources. Alternatively, the problem may be stated in terms of its dual: how to choose the most efficient method of production in order to minimize the cost of producing a given useful effect. Mises repeatedly returns to the latter formulation in his critique of socialism, with the examples of building a railway or building a house:[4] how can the socialist planners calculate the least-cost method of achieving these objects?

As regards the means for rational decision-making, Mises identifies three possible candidates: planning in kind (in natura), planning with the aid of an 'objectively recognizable unit of value' independent of market prices and money, such as labour time, and economic calculation based on market prices.

[4]The railway example is in [58]. The house-building example is in Human Action [60].

I will go on to examine Mises', very influential, arguments in section 3, but first I will examine whether an alternative interpretation can be placed on the concept of economic calculation.

It is clear that monetary calculation lends itself well to problems of the minimising or maximising sort. We can use money to find out which of several alternatives is cheaper, or which sale will yield us the most profit. But if we look in more detail at what is involved here, we shall see that a lot of calculation must take place prior to money even being brought into consideration. Let us look not at building a mere house, but at something grander, the first Pyramid at Saqqara, planned by Imhotep[4]. In order to build this Imhotep had to carry out a whole mass of calculations. He needed, for example, to know how to calculate the volume of pyramid before it was built([32], p.40), which involves a fair degree of sophisticated geometry[5]. From a knowledge of the volume of a pyramid, and a knowledge of the size of the stones he planned to use, he could calculate how many stones would be required. Knowing the rate at which stonemasons could put the stones in place he could estimate how long it would take workforces of different sizes to place all the stones for the pyramid. From the number of stones too, and knowledge of how many people are needed to transport each stone, Imhotep could work out the number of people who would have to work shifting the stone from the quarry to the pyramid.

This workforce would have to be fed, so bakers, brewers and butchers were needed to feed them ([13],ch.6). He, or his scribes, would have to calculate how many of these tradesmen were required. Quantities of grain and cattle would have to to be estimated. In the broadest sense, this was all economic calculation, but it would have taken place without money, which had yet to be invented. It might be objected that this is not what Mises meant by economic calculation, since Imhotep's calculation 'in kind' was not economic calculation but engineering calculation, a mere listing of prerequisites, what was missing was the valuation or costing of these inputs. Fair enough, this is not what von Mises meant by economic calculation, the question is, whether he was right to limit this concept to monetary calculation. Imhotep's calculations do reveal that Mises concept may have been too narrow. Suppose that the pyramid were built now, a large part of the calculations required would be the same. It would still be necessary to work out how much stone would be used, how much of various types of labour would be used, how the stone was to be transported etc. This would be the difficult part of the calculation, totaling it up in money would be easy in comparison.

Consider the issue of choosing between the most economical alternative. Imhotep certainly had to address this question. Building a pyramid was, even by modern standards, a massive undertaking. To complete it he not only had to address questions of structural stability but he also had to devise a practical method by which stones could be raised into place. That this was no easy task

[5] The Rhind Papyrus, the earliest known collection of mathematical problems, includes examples where the student had to calculate the volume of, and thus the number of bricks required for, pyramids.

is born out by the fact that we still do not know for sure how it was done. Various suggestions have been made: sloping ramps at right angles to the pyramid wall up which stones were hauled; spiral ramps wrapping round the pyramid; internal tunnel ramps; a series of manually operated cranes; etc. If we today can think of lots of possible ways in which it might be done, so to, we can assume, must the original builders, before settling on whatever method that they actually used. The resources of manpower available to them were not unlimited, so they had to discover an approach that was both technically feasible and economically feasible. This is the sort of rational choice that Mises saw as impossible without money, but the fact that the pyramids were built, indicates that some calculation of this type did occur.

The ultimate constraint here was the labour supply available; no sensible architect would embark on a course of construction that used far more labour than another. In a pre-mercantile economy like ancient Egypt this labour constraint appears directly, in a mercantile economy, the labour constraint appears indirectly in the form of monetary cost. The classical political economists argued that money relations disguised underlying relations of labour, money costs hid labour costs; money was, for Adam Smith, ultimately the power to command the labour of others.

3 Planning in kind

The organisational task that faced a pyramid architect was vast. That it was possible without money was an indication that monetary calculation was not a sine qua non of calculation. But as the project being planned becomes more complex, then planning it in material units will become more complex. Mises is in effect arguing that optimization in complex systems necessarily involves arithmetic, in the form of the explicit maximization of a scalar objective function (profit under capitalism being the paradigmatic case), and that maximising the money return on output, or minimising money cost of inputs is the only possible such scalar objective function. Mises argued for the impossibility of of planning in kind because, he said,. the human mind is limited in the degree of complexity that it can handle.

So might the employment of means other than a human mind make possible planning in kind for complex systems?

There are two 'inhuman' systems to consider:

1. Bureaucracies. A bureaucracy is made up of individual humans, but by collaborating on information processing tasks, they can carry out tasks that are impossible to one individual.

2. Computer networks. Nobody familiar with the power of Google[6] to consolidate and analyse information will need persuading that computers

[6]The algorithms used by Google involve the solution to large sparse systems of linear equations. This, as we shall see later, is the same type of calculation as is required for planning in kind. For a discussion of the linear algebra used in information retrieval see the book by Google researcher Dominic Widdows[62] or [57].

> can handle volumes and complexities of information that would stupefy a single human mind, so a computer network could clearly do economic calculations far beyond an individual human mind.

More generally as Turing pointed out [55] any extensive calculation by human beings depends on artificial aides-memoir, papyrus, clay tablets, slates, etc. With the existence of such aides to memory, algorithmic calculation becomes possible, and at this point the difference between what can be calculated by a human using paper and pencil methods or a digital computer come down only to matters of speed[53, 54]. There is thus no difference in principle between planning using a bureaucracy and planning using computers, but there is in practice a big difference in the complexity of problem that can be expeditiously handled.

There is no question that the procedure of economic calculation considered by von Mises was primarily algorithmic. It involves a fixed process of

1. For each possible technique of production
   (a) form a physical bill of materials,
   (b) use a price list to convert this into a list of money expenditures,
   (c) then add up the list to form a final cost
2. Select the cheapest final cost out of all the costs of techniques of production

We will come back to Mises's problem after looking at the views of his opponent Neurath.

3.1 Neurath's original argument.

Mises was initially debating against Otto Neurath. In an article dated 1919 Neurath had argued that a socialist economy would be able to operate calculations in-natura rather than by means of money[42], though he arguably did not provide a practical means of doing this [56]. Mises is much better known in the English speaking world than Neurath, in large part because translations of Neurath's economics works have only recently appeared. The fact that Mises's readers have not had direct access to the ideas against which Mises was arguing may have helped the plausibility of Mises's argument. It is thus worth recapitulating what Neurath meant by calculation in kind, so that one can asses to what extent Mises's criticisms were fair.

In his 1919 paper, Neurath argues that the experience of the war economy allowed one to see certain key weaknesses of past economic thought.

> Conventional economic theory mostly stands in too rigid a connection to monetary economics and has until now almost entirely neglected the in-kind economy.([42], p 300)

The war economy had in contrast been largely an in-kind economy.

> As a result of the war the in-kind calculus was applied more often and more systematically than before... It was all to apparent that war was fought with ammunition and the supply of food, not with money.([42], p304)

In kind views of quality of life. He argues that this represents a return to the original concerns of economics in the science of household economics and the science of government. Smith had been particularly concerned with the real rather than the monetary income of society, but this had been forgotten by subsequent economists who had concentrated on monetary magnitudes. Neurath advocated an explicitly Epicurean approach to economics identifying his approach as social Epicureanism. Neurath claimed that this Epicureanism lay at the basis of Marx's thought too, though if Marx's doctoral dissertation is to be believed[34], Neurath's emphasis on the empirical investigation of real conditions owes more to Democritus. If one wanted to know whether real *quality of life* of the population was improving or not one had to examine their lives in material not money terms[41]. He wrote that economics must be the study of happiness and the quality of real life. To do this economists should collect detailed statistics of the quality of life of groups in the population. These would include not only on the consumption of food, clothing and housing conditions, but also on mortality and morbidity, educational level, leisure activities, people's feelings of powerfulness versus powerlessness.

> With some expectation of success we can attempt to assemble all conditions of life into certain larger groups and arrange them according to the pleasurableness of the qualities of life caused by them. We can, for example, state what food the individuals consume per year, what their housing conditions are, what and how much they read. what their experiences are in family life, how much they work, how often and how seriously they fall ill, how much time they spend walking, attending religious services, enjoying art, etc. We can even discover certain average biographies, deviations from which appear unimportant for rough investigations. In similar ways we can also determine the conditions of life of whole groups of people by stating which proportion of them suffer from certain ailments, which proportion dies at a certain age, which proportion lives in certain homes. etc., finally even which proportion enjoys particular types of conditions of life. It is obvious that quantities which can be measured and determined clearly find more extensive treatment than the vaguer ones like religiosity, artistic activities and the like. But one must beware of thinking that all those quantities which can be treated more easily are more important, or essentially different from the vague ones. Occupational prestige, for example, is as much a part of one's income as eating and drinking. ([41] page 326)

Compared to such statistics in kind, figures for national income were, he said, far less revealing. In particular he cautions against accepting the notion of *'real income'* or inflation adjusted money income as a surrogate for the quality of life. Such 'real income' is just a reflection of money income and as such only takes into account things that are bought and sold as commodities.

> The current concept of consumption, [so-called] real income, is also understandable as derivative of money calculation. Given our own approach to economic efficiency, it seems appropriate to comprehend also :work and illness under the concept which covers food, clothing, housing, theatre visits, etc. These things, however, are not part of the [current] concept of consumption and real income, which covers only what appears as a reflection of money income. Real income [in this sense] has little significance in our approach to the study of economic efficiency. ([41] page 336)

What Neurath was saying here looks very modern. There has been increasing recognition of the inadequacy of purely monetary national income figures for judging the quality of life of a country's population (*sources???*). The UN development goals are informed by such concerns and are given in qualitative terms.... (*cite*). It is notable that this aspect of Neurath's argument for in-kind economics has been neglected by von Mises or his followers. Indeed Neurath argues that von Mises himself ultimately has recourse to the notion of an in-kind substratum of welfare against which different monetary measures of welfare must be judged. Mises recognises that monopoly reduces welfare thus:

> He (Mises) arrives at the remarkable statement: "But these, of course, are less important goods, which would not have been produced and consumed if the more pressing demands for a larger quantity of the monopolized commodity could have been satisfied. The difference between the values of these goods and the higher value of the quantity of monopoly goods not produced represents the loss in welfare which the monopoly has inflicted on the national economy."[7] We see that here Mises also arrives at a concept of wealth which obviously is divorced from money, since it is used to assess a money calculation, namely that of the monopolists. If, in the case of monopoly, according to Mises, there is a calculation of wealth by which one can judge money calculation, *then it should always be available and allow judgment on all economic processes*. ([40], page 429)

Neurath is here defending the distinction between exchange value and use value which comes from Aristotle[2, 36] and provided a key substratum of Marx's analysis of the commodity[35].

[7] Neurath is citing [59]page 389.

In kind calculation for production Neurath was adamant that a socialist economy had to be moneyless. In this, he was an orthodox follower of Marx, and as such much more radical than the Soviet government post war-communism. He repeatedly emphasizes that a socialist economy can not use just one single scalar unit in its calculations, whether this be money, labour hours or kilowatt hours. This relates both to :

1. The non-comensurability of final outcomes in terms not only of quality of life, but the quality of life of future generations;

2. The complexity of the technical constraints on production.

The emphasis on non-comensurability has its roots in his ideas on the measurement of outcomes, quality of life now and quality of life in the future:

> The 'positive quantities' of the socialist order also do not come to the same thing as the 'profit' of capitalism. Savings in coal, trees, etc., beyond amounting to savings in the displeasure of work, mean the preservation of future pleasure, a positive quantity. For instance, that coal is used nowadays for silly things is to be blamed for people freezing in the future. Still, one can only give vague estimates. Saving certain raw materials can become pointless if one discovers something new. The future figures in the balance sheets of the capitalist order only in so far as the demand is anticipated. The freezing people of the future only show up if there is already now a demand for future coal.([40], page 470)

Neurath follows Marx in accepting the use of labour vouchers as a possible means of distributing goods, provided that the community decides to do it this way, but denies that this method has anything more than a conventional significance. In particular he argues that labour time calculations are inadequate for the internal regulation of production. Labour time calculations presuppose a long time frame and an absence of natural resource constraints. If there are natural resource constraints, or short term shortages of particular equipment they can misrepresent what is potentially producible.

> How can points be assigned to individual articles of consumption? If there were natural work units and if it could be determined how many natural work units, in a "socially necessary" way, have been spent on each article of consumption, and if further it were possible to produce any amount of each article, then, under some additional conditions, each article could be assigned the number of points that represent its "work effort". [...] Let us now assume that the distribution is done through free choice of the consumers in proportion to their work. [...] some raw materials will be in short supply and thrift will necessary. If there is a great demand for articles made from these raw materials, either rationing will

> have to be introduced or the number of points for their distribution will have to be increased beyond the number representing the work spent on their production. Conversely articles in little demand will be offered for fewer points than would the work spent for their production. ([40], pp. 435-436)

These do not seem to be insuperable obstacles to the use of labour vouchers in distribution of final products. One could conceive of there being some sort of natural resource tax levied on goods whose production could not be expanded until the number of points for their distribution was equal to the work spent on their production. The proceeds of this tax could then contribute towards the labour expended providing free public services. But the point about labour values being insufficient for the internal regulation of production is correct. Instead he advocates detailed statistics on the consumption and use of each raw material and intermediate product. He proposes a system with two tables in kind for each raw material and intermediate product X

1. One table gives, in quantitative terms the output of X product, the imports and exports, and all the uses. He gives an illustration in which he shows the flows stocks and use of copper ore in Germany between 1918 and 1919.

2. Another table gives for X all the raw materials, types of labour and intermediate products that went into making it.

Accounting balances in kind will be used to check the correctness of the production and uses between these different tables. If we look at this we can see that although he presents this in terms of distinct tables, these tables record the same information as respectively to the row and column vectors of an input output matrix. The one key difference is that current western I/O matrices list all quantities in the matrix in money, whereas Neurath proposes listing them in natural units : tons, litres etc. Since the work of von Neumann (discussed below) we have become used to representing the technical structure, the in-kind flows, of the economy in matrix form. By using matrices it becomes possible to express propositions about the economy in the concise notations of the matrix and vector algebra, and to have recourse to the theorems of that algebra. But there is a big difference between constructing abstract mathematical proofs and carrying out practical economic administration.

The matrix notation of von Neumann is certainly more elegant in mathematical terms, but, as a practical tool for economic calculation, Neurath's system has great advantages. Suppose that in Germany in 1919 there were 200,000 distinct industrial products to be tracked. We know from current I/O tables that one can print a table of perhaps 80 products square on an A3 page. The complete von Neumann or Leontief style I/O matrix for 200,000 products would then run to over 6 million pages. The great bulk of these entries would be blank. To take Neurath's example of copper ore, there might be a couple of dozen copper foundries using the ore, so the copper ore row of a complete von Neumann I/O matrix, would run blank (or zeros) for thousands of pages. Neurath's usage table for copper ore, could on the other hand, be printed on a single page. The representation advocated by Neurath is actually similar to that used in modern computing when dealing with large matrices, where it is called a 'sparse matrix' representation. The advantages of this representation for computerized planning are examined in *[9]* chapter 6.

But if we stick for a moment with the matrix notation familiar to modern economists, we can understand why Neurath was so adamant that socialist calculation had to be performed in kind and could not be reduced to accounting in a single surrogate unit like labour or energy. When we do accounting in money, or in a surrogate like labour, then we add up the total cost of each column of the I/O matrix, giving us a vector of final output in money terms. *A price system thus represents an enormous destruction of information*. A matrix of technical coefficients is folded down to a vector, and in the process the real *in-natura* constraints on the economy are lost sight of. This destruction of information means that an economy that works only on the basis of the price vector must blunder around with only the most approximate grasp of reality. This of course, is exactly the opposite proposition to that advanced by Mises.

To summarise, Neurath had argued that in kind calculation was needed both to allow political deliberation on the goals of the economic plan, and to ensure the coherence of the plan. Mises has no effective reply to the first point, and concentrated his fire on the second. Mises concedes that if there is no change in technique then the sort of in-kind accounting proposed by Neurath would allow the continued operation of the socialist economy. The problem came in choosing between competing techniques. Whilst Neurath clearly believed that this was possible, he is vague about how it is to be done. He does not give a procedure or algorithm by which assessments of comparative technical efficiency can be arrived at using in-kind calculation.

The question then arises as to whether, independent of the work of Neurath, there exist *in-natura* algorithms with a function analogous to those that Mises saw as essential for economic calculation?

We will argue that subsequent authors, working in the two decades after Neurath's proposals, did in fact come up first with mathematical proofs that there exist solutions to a system of calculation in kind, and then with practical algorithms to arrive at such solutions.

3.2 von Neumann

The next two players in our drama have certain similarities. Both von Neumann and Kantorovich were mathematicians rather than economists. Their contributions to economics were just one part of a variety of research achievements. In both cases this included stints working on early nuclear weapons programs, for the US and USSR[50] respectively. At least in von Neumann's case the connection of his economic work to atomic physics was more than incidental. One of his great achievements was his mathematical formalization of quantum mechanics[61] which unified the matrix mechanics of Heisenberg with the wave mechanics of Schrodinger. His work on quantum mechanics coincided with the first draft of his economic growth model[39] given as a lecture in Princeton in 1932. In both fields he employs vector spaces and matrix operators over vector spaces, complex vector spaces in the quantum mechanical case, and real vector spaces in the growth model. Kurz and Salvadori [30]argue that his growth model has to be seen as a response to the prior work of the socialist inclined mathematician Remak[48], who worked on 'superposed prices'.

> Remak then constructs 'superposed prices' for an economic system in stationary conditions in which there are as many single-product processes of production as there are products, and each process or product is represented by a different 'person' or rather activity or industry. The amounts of the different commodities acquired by a person over a certain period of time in exchange for his or her own product are of course the amounts needed as means of production to produce this product and the amounts of consumption goods in support of the person (and his or her family), given the levels of sustenance. With an appropriate choice of units, the resulting system of 'superposed prices' can be written as
>
> $$p^T = p^T C$$
>
> where C is the augmented matrix of inputs per unit of output, and p is the vector of exchange ratios. Discussing system Remak arrived at the conclusion that there is a solution to it, which is semi-positive and unique except for a scale factor. The system refers to a kind of ideal economy with independent producers, no wage labour and hence no profits. However, in Remak's view it can also be interpreted as a socialist economic system.[30]

With Remak the mathematical links to the then emerging matrix mechanics are striking - the language of superposition, the use of a unitary matrix operator C analogous to the Hermitian operators in quantum mechanics[8]. But this apart, what is the economic significance of Remak's theory to the socialist calculation debate?

It is this. Remak shows for the first time how, starting from an *in-natura* description of the conditions of production, one can derive an equilibrium system of prices. This implies that the *in-natura* system contains the information necessary for the prices and that the prices are a projection of the *in-natura* system onto a lower dimensional space[9]. If that is the case, then any calculations that can be done with the information in the reduced system p could

[8] Like the Hermitian operators in quantum mechanics, Remak's production operator is unitary because $p$is an eigen vector of C and $|p|$ is unchanged under the operation.

[9] Suppose C is an $n \times n$ square matrix, and p an n dimensional vector. By applying Iverson's reshaping[24, 23] operator ρ, we can map C to a vector of length n^2 thus $\mathbf{c} \leftarrow (n \times n)\rho C$, and we thus see that the price system, having $n$dimensions involves a massive dimension reduction from the n^2 dimensional vector $\mathbf{c}$.

in principle be done, by some other algorithmic procedure starting from C. Remak expresses confidence that with the development of electric calculating machines, the required large systems of linear equations will be solvable.

The weakness of Remak's analysis is that it is limited to an economy in steady state. Mises had acknowledged that socialist calculation would be possible under such circumstances.

Von Neumann took the debate on in two distinct ways:

1. He models an economy in growth, not a static economy. He assumes an economy in uniform proportionate growth. He explicitly abjures considering the effects of restricted natural resources or labour supply, assuming instead that the labour supply can be extended to accommodate growth. This is perhaps not unrealistic as a picture of an economy undergoing rapid industrialization (for instance Soviet Russia at the time he was writing).

2. He allows for there to be multiple techniques to produce any given good - Remak only allowed one. These different possible productive techniques use different mixtures of inputs, and only some of them will be viable.

von Neumann again uses the idea of a technology matrix introduced by Remak, but now splits it into two matrices A which represents the goods consumed in production, and B which represents the goods produced. So a_{ij} is the amount of the j th product used in production process i, and b_{ij} the amount of product j produced in process i. This formulation allows for joint production, and he says that the depreciation of capital goods can be modeled in this way, a production process uses up new machines and produces as a side effect older, worn machines. The number of processes does not need to equal the number of distinct product types, so we are not necessarily dealing with square matrices.

Like Remak he assumes that there exists a price vector y but also an intensity vector x which measures the intensity with which any given production process is operated. We will see below that the same formulation is used by Kantorovich. Two remaining variables β and α measure the interest rate and the rate of growth of the economy respectively.

He makes two additional assumptions. First is that there are 'no profits', by which he means that all production processes with positive intensity return exactly the rate of interest. He only counts as profit, earning a return above the rate of interest. This also means that no processes are run at a loss (returning less than β). His second assumption is that any product produced in excessive quantity has a zero price.

He goes on to show that in this system there is an equilibrium state in which there is a unique growth rate $\alpha = \beta$ and definite set of intensities and prices. The intensities and prices are simultaneously determined.

What are the significant results here?

- The $in-natura$ techniques available to the economy, captured in his matrices A, B determine which processes of production should be used and in which intensities.

- They also determine an equilibrium set of prices. No system of subjective preferences is required to derive these.
- The *in-natura* techniques also determine the rate of growth and rate of interest.

What are the social relations in this model?

It is unclear. If it is a capitalist economy he is making the rather unrealistic assumption that all interest income is reinvested, so that interest becomes not so much a payment to the bank as an accounting convention. It is also unclear how a real capitalist economy could reach the equilibrium path shown. Sraffa [52] presents a rather similar model, explicitly identifying it with capitalist production, but with the crucial addition that Sraffa allows for the possibility of capitalist consumption out of interest. In the absence of any capitalist consumption, the interpretation of von Neumann's model as being of an administrative economy, is plausible. However, it is an administrative economy with at least accounting prices and a notional accounting charge for capital use. If he means that the economy is to be understood as capitalist, then he should really prove that his twin conditions of zero prices for goods in excess supply and an absolutely uniform rate of profit, can be achieved by market competition. Showing this would have been non-trivial. Indeed there is reason to suspect that uniform profit rates can not be achieved in dynamic models of this type[18].

If we suppose that von Neumann is describing an administrative economy, then it is significantly different from Neurath's idea, because of the existence of at least an administrative price vector. But this price vector is shown to arise, along with the interest rate, purely from the in-kind structure of the economy, so, as with Remak, prices are a derived sub-space. Von Neumann's paper does not, however, provide a procedure by which the equilibrium solution to the economy can be calculated. He proves the existence of such a solution but does not give a means of computing it.

If we have no joint production and only one process to produce each product, it is relatively simple to solve the VN model. Suppose we have several product types one of which is corn, with the von Neumann matrices A, B such that both are square and $B = I$. Suppose further that we have the variables in table 1, then Algorithm 1 will find the prices, growth rate, and intensities arbitrarily close to the von Neumann solution depending on ε.

If A, B take on the values given in Table 2, then with $\varepsilon = 0.001$the algorithm gives the approximate solution shown in the lower part of Table 2.

3.3 Kantorovich's method

In the early 30s, no algorithmic techniques were known which would solve the more general problem where there can be joint production and multiple possible techniques to produce individual products. But in 1939 [25] the Soviet

Table 1: Variables used in algorithm 1.

variable	meaning
x	intensity vector
n	net output vector
μ	inputs used
y	price vector denominated in corn
c	per unit cost vector in corn
β	interest rate
α	growth rate
sales	total sales in corn units
costs	total costs in corn units

Algorithm 1 Solving a VN model with no choice of techniques.

begin
  $x \leftarrow T$; — initial intensities
  $y \leftarrow 1$; — initial pirces
  $\beta \leftarrow 0.2$; — estimated interest
  repeat
    $\alpha \leftarrow \beta$;
    $c \leftarrow (A.y) \times (1 + \beta)$; — compute cost per unit
    $y \leftarrow c$; — set prices
    $y_{corn} \leftarrow 1$;
    $\mu \leftarrow \sum ((A^T) \times x)$; — compute usage
    $sales \leftarrow x.y$;
    $n \leftarrow x - \mu$;
    $costs \leftarrow y.\mu$;
    $\beta \leftarrow \frac{sales - costs}{costs}$; — recompute interest
    $x \leftarrow 0.5 \times (x + \mu \times (1 + \alpha))$;

the above line will make y move towards a composition in which the physical proportions of inputs and outputs are the same

  until $|\beta - \alpha| < \varepsilon$;

end .

Table 2: Example A and B matrices and the VN solution they give rise to.

A

corn	coal	iron
0.20	0.10	0.02
0.20	0.20	0.10
0.20	0.70	0.10

B

1.00	0.00	0.00
0.00	1.00	0.00
0.00	0.00	1.00

Solution

	corn	coal	iron
n	3.11427	3.46149	1.02518
y	1.00000	1.80357	3.56645
x	6.09637	6.88489	2.04303
$\beta =$	1.01806		
$\alpha =$	1.01866		

mathematician V Kantorovich came up with a method which later came to be known as *linear programming* or *linear optimisation,* for which he was later awarded both Stalin and Nobel prizes. Describing his discovery he wrote:

> I discovered that a whole range of problems of the most diverse character relating to the scientific organization of production (questions of the optimum distribution of the work of machines and mechanisms, the minimization of scrap, the best utilization of raw materials and local materials, fuel, transportation, and so on) lead to the formulation of a single group of mathematical problems (extremal problems). These problems are not directly comparable to problems considered in mathematical analysis. It is more correct to say that they are formally similar, and even turn out to be formally very simple, but the process of solving them with which one is faced [i.e., by mathematical analysis] is practically completely unusable, since it requires the solution of tens of thousands or even millions of systems of equations for completion.
>
> I have succeeded in finding a comparatively simple general method of solving this group of problems which is applicable to all the problems I have mentioned, and is sufficiently simple and effective for their solution to be made completely achievable under practical conditions. ([25], p. 368)

What was significant about Kantorovich's work was that he showed that it was possible, starting out from a description in purely physical terms of the various production techniques available, to use a determinate mathematical procedure to determine which combination of techniques will best meet plan targets. He

Table 3: Kantorovich's first example.

Type of machine	# machines	output per machine		Total output	
		As	Bs	As	Bs
Milling machines	3	10	20	30	60
Turret lathes	3	20	30	60	90
Automatic turret lathes	1	30	80	30	80
Max total				120	230

Table 4: Kantorovich's examples of output assignments.

Type of machine	Simple solution		Best solution	
	As	Bs	As	Bs
Milling machines	20	20	26	6
Turret lathes	36	36	60	0
Automatic turret lathes	21	21	0	80
Total	77	77	86	86

indirectly challenged von Mises[10], both by proving that in-natura calculation is possible, and by showing that there can be a non monetary scalar objective function : the degree to which plan targets are met.

The practical problems with which he was concerned came up whilst working in the plywood industry. He wanted to determine the most effective way of utilising a set of machines to maximise output. Suppose we are making a final product that requires two components, an A and a B. Altogether these must be supplied in equal numbers. We also have three types of machines whose productivities are shown in the Table 3.

Suppose we set each machine to produce equal numbers of As and Bs. The three milling machines can produce 30 As per hour or 60 Bs per hour. If the three machines produce As for 40 mins in the hour and Bs for 20 mins then they can produce 20 of each. Applying similar divisions of time we can produce 36 As and Bs on the Turret lathes and 21 As and Bs on the automatic turret lathe (Table 4).

But Kantorovich goes on to show that this assignment of machines is not the best. If we assign the automatic lathe to producing only Bs, the turret lathe to producing only As and split the time of the milling machines so that they spend 6 mins per hour producing Bs and the rest producing As, the total output per hour rises from 77 As and Bs to 86 As and Bs.

The key concept here is that each machine should be preferentially assigned to producing the part for which it is relatively most efficient. The relative efficiency of producing As/Bs of the three machines was milling machine $= \frac{1}{2}$, turret lathes $= \frac{2}{3}$, and automatic lathe $=\frac{3}{8}$. Clearly the turret lathe is relatively most efficient at producing As, the automatic lathe is relatively most efficient at producing Bs and the milling machine stands in between. Thus the automatic lathe is set to produce only Bs, the turret lathes to make only As and the time of the milling machines is split so as to ensure that an equal number of each

[10]There is no indication that he was aware of von Mises at the time.

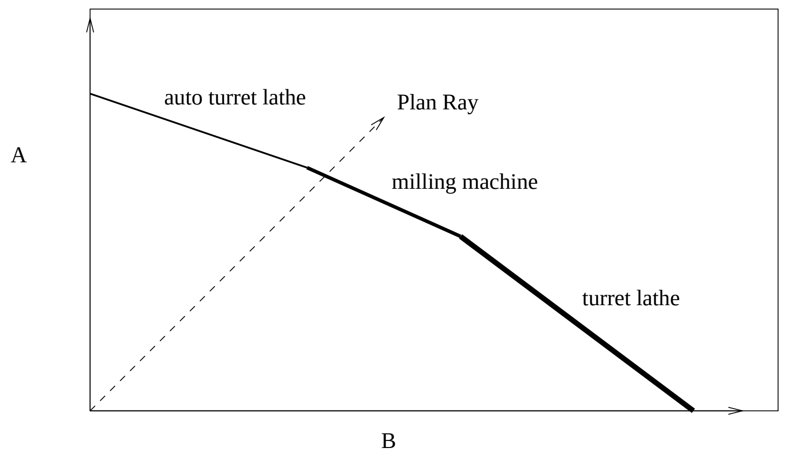

Figure 1: Kantorovich's example as a diagram. The plan ray is the locus all points where the output of As equals the output of Bs. The production possibility frontier is made of straight line segments whose slopes represent the relative productivities of the various machines for the two products. As a whole these make a polygon. The plan objective is best met where the plan ray intersects the boundary of this polygon.

product is turned out.

The decision process is shown diagrammatically in Figure 1. The key to the construction of the diagram, and to the decision algorithm is to rank the machines in order of their relative productivities. If one does this, one obtains a convex polygon whose line segments represent the different machines. The slopes of the line segments are the relative productivities of the machines. One starts out on the left with the machine that is relatively best at producing Bs, then move through the machines in descending order of relative productivity. Because relative productivity is monotonically decreasing one is guaranteed that the boundary will be convex. One then computes the intersection of the 45 degree line representing equal output of As and Bs with the boundary of this polygon. This intersection point is the optimal way of meeting the plan. The term linear programming stems from the fact that the production functions are represented by straight lines in the case of 2 products, planes for 3 products, and for the general higher dimensional case by linear functions. That is to say, functions in which variables only appear raised to the power 1.

The slope of the boundary where the plan ray intersects was called by Kantorovich the resolving ratio. Any machine whose slope is less than this should be assigned to produce Bs any machine whose slope is greater, should be assigned to produce As.

When there are only two products being considered, the method is easy and lends itself to diagrammatic representation. But it can handle problems of higher dimensions, involving 3 or more products. In these cases we can not use graphical solutions, but Kantorovich provided an algorithmic by which the resolving ratios for different pairs of outputs could be arrived at by successive approximations. Kantorovich's work was unknown outside of the USSR until the late 50s and prior to that Dantzig had independently developed a sim-

Algorithm 2 Kantorovich's example as equations input to `lp_solve`..

```
A;
m1<=3;
m2<=3;
m3<=1;
A-B=0;
m1-0.1 x1a - 0.05 x1b=0;
m2-0.05 x2a - 0.033333 x2b=0;
m3- 0.033333 x3a - 0.0125 x3b=0;
x1a+x2a+x3a - A=0;
x1b+x2b+x3b -B =0;
int A;
```

ilar algorithm for solving linear programming problems, the so called simplex method [12]. This has subsequently been incorporated into freely available software tools[11]. These packages allow you to enter the problem as a set of linear equations or linear inequalities which they then solve.

In the West, linear programming was used to optimise the use of production facilities operating within a capitalist market. This meant that the objective function that was maximised was not a fixed mix of outputs, in Kantorovich's first example equal numbers of parts A and B, but the money that would be obtained from selling the output: price A $\times$number of As + price B $\times$number of Bs. Manuals and textbooks produced in association with Western linear programming software assumes this sort of objective. However, as we shall see, one can readily formulate Kantorovich's problem using this sort of software by adding additional equations. We shall now show how you can use the package `lp_solve` to reproduce Kantorovich's solution to his problem.

The program requires that you input an expression to be maximised or minimised followed by a sequence of equations or inequalities. In Algorithm 2 we give Kantorovich's problem in the format that `lp_solve` requires. In this example we use the following variables:

variable	meaning
A	number of units of A produced
B	number of units of B produced
m1	number of milling machines used
m2	number of turret lathes used
m3	number of automatic turret lathes used
xij	number of units of j produced on machine i

Thus x1a means the output of As on milling machines.

The first line of input is the objective function to be maximised. We give this as A, meaning maximise the output of A's. The following lines give the constraints to which the maximisation process is to be subjected.

`A-B=0`

This is another way of writing that A=B, or that equal quantities

[11] For example `lp_solve` and `GLPK`.

of A and B must be produced.

```
m1<=3
```

This means that the number of milling machines used must be less than or equal to 3. The characters '<' '=' are used because $\leq$ is not available on computer keyboards. Similar constraints are provided for the other machines.

```
m1-0.1x1a-0.05x1b=0
```

This specifies $m1 = 0.1x1a + 0.05x1b = \frac{1}{10}x1a + \frac{1}{20}x1b$ or in words, that allocating a milling machine to produce an A uses $\frac{1}{10}$ of a milling machine hour, and that allocating a milling machine to produce a unit of B uses $\frac{1}{20}$ of a milling machine hour. We provide similar production equations for the other machines.

```
x1a+x2a+x3a - A=0
```

This says that the total output of A is equal to the sum of the outputs of A from each of the machines. We provide a similar equation defining the output of B.

Note that all equations have to be provided with variables and constants on the left and a constant on the right. One can readily re-arrange the equations in this form. The last line specifies that the number of units of A produced should be an integer. When the equations are input to lp_solve it produces the answer:

```
Value of objective function: 86
Actual values of the variables:
A           86
B           86
x1a          26
x1b          6
x2a          60
x2b          0
x3a          0
x3b          80
m1           2.9
m2           3
m3           1
```

which exactly reproduces Kantorovich's own solution (Table 4) arrived at using his algorithm.

3.4 Generalising Kantorovich's approach

In his first example Kantorovich deals with a very simple problem, producing two goods in equal proportions using a small set of machines. He was aware, even in 1939 that the potential applications of mathematical planning were much wider. We will look at two issues that he considered which are important for the more general application of the method.

1. Producing outputs in a definite ratio rather than in strictly equal quantities.

2. Taking into account consumption of raw materials and other inputs.

Suppose that instead of wanting to produce one unit of A for every unit of B, as might be the case if we were matching car engines to car bodies, we want to produce 4 units of A for every unit of B, as would be the case if we were matching wheels to car engines (and ignoring spare wheels). Can Kantorovich's method deal with this as well. Consider Figure 1 again. In that the plan ray is shown at an angle of 45° a slope of 1 to 1. If we drew the plan ray at a slope of 4 to 1, the intersection with the production frontier would provide the solution. Since this geometric approach only works for two products, let us consider the algebraic implications.

You should now be convinced that it is possible to solve Kantorovich's original problem[12] by algebraic means. In Algorithm 2 we specified that $A - B = 0$ or in other words $A = B$, if one wanted 4 units of A for every B we would have to specify $A = 4B$ or, expressing it in the standard form used in linear optimisation, $A - 4B = 0$. Suppose A stands for engines, B stands for wheels. If we now say wheels come in packs of 4, then we can repose the problem in terms of producing equal numbers of packs of wheels and engines. Introduce a new variable $\beta = 4B$ to stand for packs of wheels, and rewrite the equations in terms of β and we can return to an equation specifying the output mix in the form $A - \beta = 0$, which we know to be soluble.

How do we deal with consumption of raw materials or intermediate products?

In our previous example we had variables like `x1b` which stood for the output of product B on machine 1. This was always a positive quantity. Suppose that there is a third good to be considered - electricity, and that each machine consumes electricity at different rate depending on what it is turning out. Call electricity C and introduce new variables `x1ac, x1bc` etc referring to how much electricity is consumed by machine 1 producing outputs A and B. Then add equations specifying how much electricity is consumed by each machine doing each task, and the model will specify the total amount of electricity consumed.

We now know how to :

1. Use Kantorovich's approach to specify that outputs must be produced in a definite ratios.

2. Use it to take into account consumption of raw materials and other inputs.

[12] Actually this was his "problem A"

If we can do these two tasks, we can in principle perform *in-natura* calculations for an entire planned economy. Given a final output bundle of consumer and investment goods to maximise and given our current resources, a system of linear equations and inequalities can be solved to yield the structure of the plan. From simple beginnings, optimising the output of plywood on different machines, Kantorovich had come up with a mathematical approach which could be extended to the problem of optimising the operation of the economy as a whole.

3.5 A second example

Let us consider a more complicated example, where we have to draw up a plan for a simple economy. We imagine an economy that produces three outputs : energy, food, and machines. The production uses labour, wind and river power, and two types of land: fertile valley land, and poorer highlands. If we build dams to tap hydro power, some fertile land is flooded. Wind power on the other hand, can be produced on hilly land without compromising its use for agriculture. We want to draw up a plan that will make the most rational use of our scarce resources of people, rivers and land.

In order to plan rationally, we must know what the composition of the final output is to be - Kantorovich's ray. For simplicity we will assume that final consumption is to be made up of food and energy, and that we want to consume these in the ratio 3 units of food per unit of energy. We also need to provide equations relating to the productivities of our various technologies and the total resources available to us.

Valleys are more fertile. When we grow food in a valleys, each valley requires 10,000 workers and 1000 machines and 20,000 units of energy to produce 50,000 units of food. If we grow food on high land, then each area of high land produces only 20,000 units of food using 10,000 workers, 800 machines and 10,000 units of energy.

Electricity can be produced in two ways. A dam produces 60000 units of energy, using one valley and 100 workers and 80 machines. A windmill produces 500 units of electricity, using 4 workers and 6 machines, but the land on which it is sited can still be used for farming.

We will assume that machine production uses 20 units of electricity and 10 workers per machine produced.

Finally we are constrained by the total workforce, which we shall assume to be 104,000 people.

Tables 5 and 6 show how to express the constraints on the economy and the plan in equational form. If we feed these into `lp_solve` we obtain the plan shown in Table 7. The equation solver shows that the plan targets can best be met by building no dams, generating all electricity using 541 windmills, and devoting the river valleys to agriculture.

It also shows how labour should be best allocated between activities: 40000 people should be employed in agriculture in the valleys, 109 people should

Table 5: Variables in the example economy

e	total energy output
e_c	household energy consumption
f	food
v	valleys
w	windmills
m	machines
d	dams
u	undammed valleys
h	highland
f_h	food produced on high land
f_v	food produced in valleys

Table 6: Resource constraints and productivities in our example economy

final output mix	$f = 3e_c$
number of valleys	$v = 4$
dams use valleys	$v - u = d$
valley food output	$f_v = 50000u$
valley farm labour	$l_v = 10000u$
valley energy use	$e_v = 20000u$
valley farm machines	$m_v = 1000u$
highland food prod	$f_h = 20000h$
highland farm labour	$l_h = 10000h$
highland energy use	$e_h = 10000h$
highland farm machines	$m_h = 800h$
energy production	$e = 500w + 60000d$
energy workers	$l_e = 100d + 4w$
machines in energy prod	$m_e = 80d + 6w$
workers making machines	$l_m = 10m$
energy used to make machines	$e_m = 20m$
energy consumption	$e_m + e_v + e_h + e_c \leq e$
machine use	$m_e + m_h + m_v \leq m$
total food prod	$f = f_h + f_v$
workforce	$l_m + l_e + l_v + l_h \leq 104000$

Table 7: Economic plan for the example economy using lp_solve

d (dams)	0
e	270500
f	200218
h	0.0108889
m	6172.71
u	4
v	4
w (windmills)	541
e_c	66739.3
e_h	108.889
e_m	123454
e_v	80000
f_h	217.778
f_v	200000
l_e	2164
l_h	108.889
l_m	61727.1
l_v	40000
m_e	2164
m_h	8.71111
m_v	4000

work as farmers in the highlands, 2164 people should work on energy production, and 61727 people should work building machines.

The results that we have obtained were by no means obvious at the outset. It was not initially clear that it would be better to use all the river valleys for agriculture rather than building dams on some of them. In fact, whether dams or windmills are preferred turns out to depend on the whole system, not just on their individual rates of producing electricity. We can illustrate this by considering what happens if we cut the labour supply in half to 52000 people?

If we feed this constraint in to the system of equations we find the optimal use of resources has changed. The plan now involves 1 dam and 159 windmills. Cut the working population slightly further, down to 50000 people and the optimal plan involves flooding two valleys with dams and building only 23 windmills. Why?

As the population is reduced, there are no longer enough people available to both farm the valleys and produce agricultural machinery. Under these circumstances the higher fertility of lowland valleys is of no importance, it is better to use one or more of them to generate electricity. By applying Kantorovich's approach it becomes possible for a socialist plan to do two things that von Mises had believed impossible:

1. It allows the plan to take into account natural resource constraints - in this case the shortage of land in river valleys which can be put to alternative uses.

2. It allows rational choices to be made between different technologies - in

this case between windmills and hydro power and between lowland and highland agriculture.

Contrary to what von Mises claimed, the whole calculation can be done in physical units without any recourse to money or to prices.

4 Valuation

The core of Mises's argument relates to the use of prices to arrive at a rational use of intermediate or capital goods. Mises argues that, in practice, only money prices will do for this, but concedes that, in principle, other systems of valuation, such as labour values would also be applicable. Kantorovich too, was very concerned with the problem of relative valuation[26], and developed what he called *objectively determined valuations* (ODV). These valuations differed from prices, since a price involves an exchange of commodities for money between two owners. In the USSR all factories and all products were owned by the state. When products moved from one factory to another, there was no sale or purchase involved. The ODVs were purely notional numbers, used in economic calculations, not selling prices.

He considered a situation where planners have to deal with several different types of factories ($A..E$) each able to produce products 1 and 2, and where the intended ratio of output of product 1 and 2 are fixed in the plan. Each class of factory $A..E$ has different relative productivities for the two products.

He next looked at the apparent profitability of producing products 1 and 2 under different relative valuations. Under some schemes of relative price, all factories would find product 1 to be unprofitable relative to product 2, under other the reverse would occur. Intermediate price schemes would allow both products to be produced, with some classes of factories specializing on 1 and others on 2. He gives the example of children's clothing as something which, under the administratively determined valuations then used in the USSR, were unprofitable to produce, and unless factories were specifically instructed to ignore local profitability, too few children's clothes would be made.

He asks if there exists a relative valuation structure which would allow factories to concentrate on the most valuable output, and at the same time meet the specified plan targets and arrives at certain conclusions:

1. That among the very large number of possible plans there is always an optimal one which maximises output of plan goals with current resources.
2. That in the optimal plan there exists a set *objectively determined valuations* (ODV) of goods which will ensure that each factory
    (a) produces the output which will contribute most to maximising the plan goals
    (b) each factory also finds that the output which contributes to maximising plan targets is also the output which is most profitable

3. With arbitrary valuations which differ from ODV, these conditions can not be met, and profit maximising factories will not specialise in a way that meets plan goals optimally.

It is important to understand that his ODVs are valuations that apply only for a plan which optimally meets a specific plan target. Kantorovich's procedure for arriving at an optimal plan involved successive adjustments to the ODVs and factory specialisation until both the appropriate mix of goods is reached, and at the same time each factory is producing its most profitable good. He actually gave several different mathematical procedures for arriving at such a plan and system of ODVs.

Although Kantorovich asserts that labour is ultimately the only source of value, his ODV's are short term valuations and differ from the classical labour theory of value, which gave valuations in terms of the long term labour reproduction costs of goods - including the reproduction costs of capital goods. Kantorovich, in contrast, is concerned with valuations which should apply with the current stock of means of production and labour resources. For example, he considers the situation of giving a valuation to electric power relative to labour. Instead of valuing it in terms of the labour required to produce electricity, he first assumes that the total electrical power available is fixed - ie, power-stations operating at full capacity, and then works out how many person hours of labour is saved by using an additional kilowatt hour of electricity. The assumption is made that in order to arrive at this objective valuation of electricity in terms of labour

1. The plan targets must be met

2. The plan must be optimal

Kantorovich's insistence on considering short term, very material constraints - so many megawatts of power, such and such a number of cutting machines, etc, gives his work an intensely practical and pragmatic character, quite different from that of most theoretical economists.

Why is Kantorovich so concerned with valuations and profitability?

There seem to be two reasons. We should first note that by profit maximising Kantorovich actually meant maximising the value of output. This must be understood in the context of Soviet practice where mines and factories were given incentives to over-fulfill plan targets. If the output was a single good - say coal, the target could be specified in tons. But if the factory produced several goods, then the target had to be set in terms of x rubles worth of a mix of goods. With the 'wrong' price structure, plants would attempt to maximise the production of the goods which were of the highest value, ignoring those of lower value, with the result that the aggregate supply of all goods was often not in the proportions that the planners intended. This practice of setting plan targets in money terms reflected the limited ability of GOS PLAN to specify detailed targets in kind as described by [43].

The second reason relates to his particular algorithm for solving linear programming problems which used an iterative adjustments to initial ODVs until an optimal plan is achieved.

These two aspects seem intimately linked in his presentation, but the presuppositions about the incentives to factories are not brought to the fore.

With computer algorithms, the process of solving a linear program becomes a 'black box'. The user need not concern themselves with details such as the method of calculation - whether it uses Kantorovich's approach Danzig's or Karmarkar's, except insofar as this affects the size of problem that can be handled, as we discuss in section 5. With computer packages, ODVs would no longer be needed for computing a plan, but would they still be needed for specifying targets to factories?

This depends on the information processing capacity of the planning system. If it were capable of specifying fully disaggregated plans, then it could in principle just place orders with factories for specific quantities of each good. In these circumstances, the factories could not cheat by producing more of high value items and less of low value ones. Indeed, the very information that would be required to compute Kantorovich's ODVs, would have been sufficient for GOS PLAN to specify disaggregated orders in kind for the products that would have had valuations attached.

There remains another level at which valuations would have been useful - when product designs were being drawn up at a local level. If a refrigerator designer was deciding on what components to use in a planned new model, she would need some way of telling which components would, from a social standpoint, have been the most economical, which implies a system of valuations. However it is not clear that the full apparatus of ODVs would be either necessary or appropriate here. ODV's correspond to a system of marginal cost, rather than average cost pricing. They reflect current marginal costs with the immediately current constraints on production. The use of such marginal costing was criticized by other Soviet economists[22, 37]. It is not clear, in retrospect, that they would have been more appropriate than a system of average cost valuation if one was projecting ahead a year or so. Indeed, given the stochastic properties of prices in a real capitalist economy[19], it is doubtful that, with the exception of certain constrained products like oil, the difference between average and marginal costs is significant in the west.

5 Complexity

Linear programming, originally pioneered by Kantorovich, provides an answer in principle to von Mises claim that rational economic calculation is impossible without money. But this is an answer only in principle. Linear programming would only be a practical solution to the problem if it were possible, in practice, to solve the equations required in a socialist plan. This in turn requires the existence of a practical algorithm for solving them, and sufficient computational resources to implement the algorithm. Kantorovich, in an appendix to [25], gave a practical algorithm, to be executed by paper and pencil mathematics. The algorithm was sufficiently tractable for these techniques to be used to solve practical problems of a modest scale. When tackling larger problems he advised the use of approximative techniques like aggregating similar production processes and treating them as a single composite process. Whilst Kantorovich's algorithm uses his ODV's, which he earlier called *resolving multipliers*, subsequent algorithms for linear programming do not, so the ODVs should not be considered as fundamental to the field.

Since the pioneering work on linear programming in the 30s, computing has been transformed from something done by human 'computors' to something done by electronic ones. The speed at which calculations can be done has increased many billion-fold. It is now possible to use software packages to solve huge systems of linear equations. But are computers powerful enough to be used to plan an entire economy?

In a large economy like the former USSR there were probably several million distinct types of industrial products, ranging from the various sorts of screws, washers and types of electronic components to large final products like ships and airliners. Although there was great enthusiasm for Kantorovich's methods in the USSR during the 60s, the scale of the economy was to great for his techniques to be used for detailed planning with the then available computer technology. Instead they were used either in optimising particular production plants, or, in drawing up aggregated sectoral plans for the economy as a whole. How has the situation changed today, given that the power of computers has continued to grow at an exponential rate since the fall of the USSR?

5.1 Complexity classes

To answer this one needs to be able to quantify the complexity of a planning task and compare it with available computing resources. Measuring complexity is a branch of algorithmics. Algorithms are classified into complexity classes. For instance, computing the average of a list of n numbers is said to be of complexity class n, because the number of simple arithmetic operations required will be proportional to n. This complexity class is termed linear, as the algorithms execution time on a computer grows linearly with the number of items.

A bit more complex than linear algorithms are the log-linear ones. It turns out that one can sort a list of n numbers into ascending order using $n\log(n)$ basic arithmetic operations. Problems which are either linear or log-linear are reckoned to be very easy to solve on computers.

Next in difficulty come the polynomial problems where the number of basic arithmetic steps grows as some polynomial function of the size of the input data. If an algorithm had a running time that was proportional to n^2 or to $n^3$for some size of input data n then it would be of polynomial complexity. In algorithmics, polynomial problems are regarded as being tractable, since, with computers able to do billions of operations per second, such problems can be solved for quite large values of n. For example multiplication is a task that grows polynomially with the number of digits in the numbers. If you want to multiply 17 by 32 you have to carry out the basic steps $2 \times 7 = 14$, $2 \times 10 = 20$, $30 \times 7 = 210$, $30 \times 10 = 300$ and then add up the partial products. The number of multiplication steps will grow as n^2, where n is the number of digits in the numbers.

Beyond the polynomial problems comes the class of NP or non-deterministic polynomial problems. These are problems which, were you to take them to Oracle at Delphi, and were the priestess to give you an answer, you could check whether her answer was correct in polynomial time. Suppose you had a 100 digit number x and asked the priestess what its prime factors were, and she replied with one 47 digit number and one 53 digit number. You could take this on trust, or bearing in mind the many tales of those mislead by the Divine Oracle, you might decide to check if her answer was correct. If she were right, then multiplying the two numbers she gave you should yield x. This multiplication would take you of the order of $47 \times 53 = 2491$ basic operations, which is roughly $\frac{1}{4}n^2$ in terms of the length of the original number you gave the priestess. This shows that we can check the validity of purported prime factors in polynomial time.

Sadly, the Oracle at Delphi has long fallen from use, and we, lacking that divine guidance once available to the Ancients, must find prime factors by mundane means. A mundane and deterministic procedure is to test all prime $y \in 2..\sqrt{x}$ to see if $\frac{x}{y}$ is a whole number. The first such y is a prime factor. The drawback is the vast number of tests that must be performed. For 100 digit numbers we would have to test all y in the range $2..10^{50}$ to be sure of finding a prime factor if one existed. The number of tests to be performed grows as $10^{\frac{n}{2}}$, in other words the number of tests grows exponentially with n. This problem, and others in the class of exponential problems, is assumed to be computationally intractable, since the number of possibilities to be checked grows so rapidly that it rapidly exhausts the power of even the swiftest computer. Indeed so hard is the task that certain cryptographic protocols[49] rely on large prime factors being practically impossible to discover.

5.2 Complexity class of economic planning

After that short introduction to the idea of complexity classes, let us apply these ideas to economic planning. To what complexity class does linear programming belong?

> For a long time it was not known whether or not linear programs belonged to a non-polynomial class called "hard" (such as the one the traveling salesman problem belongs to) or to an "easy" polynomial class (like the one that the shortest path problem belongs to). In 1970, Victor Klee[29] and George Minty created an example that showed that the classical simplex algorithm would require an exponential number of steps to solve a worst-case linear program. In 1978, the Russian mathematician L. G.

> Khachian[28] developed a polynomial-time algorithm for solving linear programs. It is an interior method using ellipsoids inscribed in the feasible region. He proved that the computing time is guaranteed to be less that a polynomial expression in the dimensions of the problem and the number of digits of input data. Although polynomial, the bound he established turned out to be too high for his algorithm to be used to solve practical problems.
>
> Karmarkar's algorithm [27] was an important improvement on the theoretical result of Khachian that a showed how linear program can be solved in polynomial time. Moreover his algorithm turned out to be one which could be used to solve practical linear programs. (*Dantzig* [11])

Modern linear programming packages tend to combine Dantzig's simplex method with the more recent interior point methods. This allows the most modern implementations to solve programming problems involving up to one billion variables[21, 20]. For such huge problems large parallel supercomputers with over a thousand processor chips are used. But even with much more modest 4 CPU computers, linear programming problems in the million variable class were being solved in half an hour using interior point methods[13].

These advances in linear programming algorithms and in computer technology mean that linear programming could now be applied to detailed planning at the whole economy level, rather than just at an aggregate level.

6 Deriving the plan ray

Kantorovich assumed that the plan had a given target to optimise in the form of a particular mix of goods: the plan ray. This reflected the social reality for those engaged in managing Soviet industry, in that they were given a mix of products to produce by GOS PLAN. The planning authorities themselves however, needed to specify what this ultimate output mix would be. In the early phases of Soviet planning, when Kantorovich wrote his original paper, the goals set by the planners were primarily directed at achieving rapid industrialization and building up a defence base against the threat of invasion. The planning process was successful in achieving these goals. But in an already industrialised country, in times of peace, the meeting of current social needs becomes the first priority and so plan vector has to be pointed in that direction. A criticism commonly leveled at the Soviet-type economies—and not only by their Western detractors—is that they were unresponsive to consumer demand. It is therefore important to our general argument to demonstrate that a planned economy can be responsive to the changing pattern of consumer preferences—that the shortages, queues and surpluses of unwanted goods of which we hear so much are not an inherent feature of socialist planning. The economists Dickinson and

[13]See[3] chap 4. The Harmony Algorithm for constructing plans, given in [9], is an instance of the class of algorithm discussed by Bienstock.

Lange, writing just prior to Kantorovich, outlined a practical mechanism by which this could be done [31, 16].

They proposed that the state wholesale sector should operate on a break-even basis with flexible prices. Wholesale managers would set market clearing prices for the products on sale as consumer goods. These wholesale prices would then act as a guide to the plan authorities, telling them whether to increase or decrease production of particular lines of product. If prices were high, then that line of product would have its output increased, otherwise its planned output would be reduced.

The basic idea is clear, the same principle that adjusts production of consumer goods in a capitalist economy was to be employed. But this then raises the problem of how one determines that a price is high or low. High or low relative to what?

What would be the basis of valuation used?

After incorrectly rejecting the possibility of planning in kind, Mises had considered the possibility that the socialist planners might be able to make use of an 'objectively recognizable unity of value', i.e., some measurable property of goods, in performing their economic calculations. The only candidate Mises could see for such a unit is labour content, as in the theories of value of Ricardo and Marx. The latter had proposed that workers be paid in labour tokens and that goods be priced similarly[33]. Mises ended up rejecting labour as a value unit; he had two relevant arguments, each purporting to show that labour content cannot provide an adequate measure of the cost of production. These arguments concern the neglect of natural resource costs implicit in the use of labour values, and the inhomogeneity of labour. Mises's critique of labour values is very brief and sketchy. Two pages or so of substantive argument appear in [58] and are reproduced in [60]. This doubtless reflects the fact that although Marx and Engels had laid great stress on planning as an allocation of labour time, this conception had been more or less abandoned by English speaking socialist economists by the late 30s. Neither Lange nor Dickinson relied on the classical theory of value in their arguments. Writing in 1930, Appel[1] had laid great stress on the relevance of the labour theory of value for socialist economics, but his ideas were largely ignored. More recent writers have again laid emphasis on Marx's theory of value as a guide to socialist planning[17, 46, 45, 9].

The basic principle in these schemes can be stated quite simply. All consumer goods are marked with their labour values, i.e. the total amount of social labour which is required to produce them, both directly and indirectly. But aside from this, the actual prices (in labour tokens) of consumer goods will be set, as far as possible, at market-clearing levels. Suppose a particular item requires 10 hours of labour to produce. It will then be marked with a labour value of 10 hours, but if an excess demand for the item emerges when it is priced at 10 labour tokens, the price will be raised so as to (approximately) eliminate the excess demand. Suppose this price happens to be 12 labour tokens. This product then has a ratio of market-clearing price to labour-value of 12/10, or 1.20. The planners record this ratio for each consumer good. We would expect the ratio to vary from product to product, sometimes around 1.0, sometimes above (if the product is in strong demand), and sometimes below (if the product is relatively unpopular). The planners then follow this rule: Increase the target output of goods with a ratio in excess of 1.0, and reduce the target for goods with a ratio less than 1.0.

The point is that these ratios provide a measure of the effectiveness of social labour in meeting consumer's needs (production of 'use-value', in Marx's terminology) across the different industries. If a product has a ratio of market-clearing price to labour-value above 1.0, this indicates that people are willing to spend more labour tokens on the item (i.e. work more hours to acquire it) than the labour time required to produce it. But this in turn indicates that the labour devoted to producing this product is of above-average 'social effectiveness'. Conversely, if the market-clearing price falls below the labour-value, that tells us that consumers do not 'value' the product at its full labour content: labour devoted to this good is of below-average effectiveness. Parity, or a ratio of 1.0, is an equilibrium condition: in this case consumers 'value' the product, in terms of their own labour time, at just what it costs society to produce it.

The feasibility of using labour time for expressing prices depends on being able to calculate it. This might seem a daunting task, but it actually involves solving a similar, though somewhat easier, set of linear equations to those required when one draws up a consistent plan. The task is thus computationally tractable on the grounds explained earlier.

Mises objected that "the ... defect in calculation in terms of labour is the ignoring of the different qualities of labour" (1935: 114). Mises notes Marx's claim that skilled labour counts as a multiple of, and hence may be reduced to, 'simple labour', but argues that there is no way to effect this reduction short of the comparison of the products of different labours in the process of market exchange. Wage differentials might appear to offer a solution, but the equalizing process in this case "is a result of market transactions and not its antecedent." Mises assumes that the socialist society will operate an egalitarian incomes policy, so that market-determined wage rates will not be available as a guide to calculation. The conclusion is then that "calculation in terms of labour would have to set up an arbitrary proportion for the substitution of complex by simple labour, which excludes its employment for purposes of economic administration" (1935: 115).

True, labour is not homogeneous, but there is no warrant for the claim that the reduction factor for complex labour has to be arbitrary under socialism. There are two possible approaches:

1. Skilled labour may be treated in the same way that Marx treats the means of production in Capital, namely as a produced input which 'transfers' embodied labour to its product over time. Given the labour time required to produce skills and a depreciation horizon for those skills, one may calculate an implied 'rate of transfer' of the labour time embodied in the skills. If we call this rate, for skill i, r_i , then labour of this type

should be counted as a multiple (1+r_i) of simple labour, for the purpose of 'costing' its products. An iterative procedure is needed: first calculate the transfer rates as if all inputs were simple labour, then use those first-round transfer rates to re-evaluate the skilled labour inputs, on this basis recompute the transfer rates, and so on, until convergence is reached.

2. Alternatively one may use the approach advocated by Kantorovich[26](page 64..66) where he shows that skilled labour of different grades can be assigned ODVs on the basis of their different productivities.

Which method is used would depend on the timescale of the calculation. If one wants short term answers to the relative valuations of different labours, then Kantorovich's approach is relevant. For longer term considerations, within the time scale that newly trained staff can be brought up to speed, then the first alternative would be appropriate.

7 Conclusion

The Soviet mathematical school founded by Kantorovich and the Austrian school exemplified by Mises and Hayek took radically different positions on the feasibility of socialist economic calculation. To a large extent they ignored one another. The Austrian school largely concentrated on criticising Western trained socialist economists like Lange and the Soviet school appears to have ignored Mises completely. Even when the key participants met, the issue was not raised. Menshikov writes:

> It is interesting that in the account of his trip to Sweden for receiving the Nobel Prize, Kantorovich mentions an informal reception with the participation of several American economists – Nobel Prize laureates – including Hayek, Leontief, and Samuelson. But, apparently, neither at this reception, nor during other meetings, this issue was never raised. In January 1976, when I worked in USA as the Director of the United Nation Projections and Perspective Studies Branch, I was asked to present L. V. Kantorovich as a new Nobel Prize laureate at the annual meeting of the American Economic Association in Atlantic City. Of course, I put the emphasis on the economic discovery of the laureate. In the discussion, none of the audience, which included T. Koopmans and L. Klein, a future Nobel Prize laureate, mentioned the question of actual Kantorovich's answer to a part of Hayek's argumentation.[37]

With the political demise of the USSR, the Austrian school have tended to assume that Mises arguments have been vindicated, but theoretical economic arguments are not finally resolved by politics. Political fashions change. Socialism, from being politically unpopular in Europe the 1990s, has, since then, been making substantial inroads on another continent. No, one has to bring economic arguments head to head in their own terms. Kantorovich, an absent participant in the Western debate on socialist calculation, is worth paying attention to.

References

[1] J. Appel, M. Baker, A.A. Deutschlands, and G.I. Kommunisten. *Fundamental Principles of Communist Production and Distribution*. Movement for Workers' Councils, von der Kollektivarbeit der Gruppe Internationaler Kommunisten - GIK [Allgemeine Arbeiter Union Deutschlands - AAUD], 1990 (1930).

[2] Aristotle. *The Politics*. Hutchinson, 1988.

[3] D. Bienstock. *Potential function methods for approximately solving linear programming problems: theory and practice*. Kluwer Academic Publishers, 2002.

[4] M.L. Bierbrier. *The Tomb-Builders of the Pharaohs*. American Univ in Cairo Press, 1989.

[5] Paul Cockshott and Allin Cottrell. Information and economics : a critique of hayek. *Research in Political Economy*, 18(1):177–202, 1997.

[6] W. P. Cockshott and A. F. Cottrell. Labour time versus alternative value bases: a research note. *Cambridge Journal of Economics*, 21:545–549, 1997.

[7] W. P. Cockshott and A. F. Cottrell. A note on the organic composition of capital and profit rates. *Cambridge Journal of Economics*, 27:749–754, 2003.

[8] W.P. Cockshott and A. Cottrell. The Scientific Status of the Labour Theory of Value. *IWGVT conference at the Eastern Economic Association meeting, in April*, 1997.

[9] Allin Cottrell and Paul Cockshott. *Towards a New Socialism*, volume Nottingham. Bertrand Russell Press, 1992.

[10] Allin Cottrell and Paul Cockshott. Calculation complexity and planning : the socialist calculation debate once again. *Review of Political Economy*, 5(1):73–112, January 1993.

[11] G Dantzig. Linear Programming. *Operations Research*, 50(1):42–47, 2002.

[12] G. B. Dantzig. *Linear Programming and Extensions*. Princeton University Press, Princeton, 1963.

[13] A.R. David. *The Pyramid Builders of Ancient Egypt:: a Modern Investigation of Pharaoh's Workforce*. Routledge, 1996.

[14] R. Dawkins. *The God Delusion*. Houghton Mifflin Co., 2006.

[15] R. Dawkins et al. *Unweaving the Rainbow: Science, Delusion and the Appetite for Wonder*. Houghton Mifflin Company, 1998.

[16] HD Dickinson. Price Formation in a Socialist Community. *The Economic Journal*, 43(170):237–250, 1933.

[17] H. Dieterich. La democracia participativa: el socialismo del siglo XXI, País. *Vasco, Baigorri*, 2002.

[18] Michel-Stephane Dupertuis and Ajit Sinha. A sraffian critique of the classical notion of center of gravitation. Technical report.

[19] Emmanuel Farjoun and Moshe Machover. *Laws of Chaos, a Probabilistic Approach to Political Economy*. Verso, London, 1983.

[20] Jacek Gondzio and Andreas Grothey. Massively parallel implementation of interior point methods for very large scale optimization. In R. Wyrzykowski, J. Dongarra, N. Meyer, and J. Wasniewski, editors, *Parallel Processing and Applied Mathematics, Lecture Notes in Computer Science, 3911*, Berlin, September 2006. Springer-Verlag.

[21] Jacek Gondzio and Andreas Grothey. Solving nonlinear financial planning problems with 10^9 decision variables on massively parallel architectures. In C. A. Brebia, editor, *Computational Finance and its Applications II, WIT Transactions on Modelling and Simulation*, volume 43. WIT Press, 2006.

[22] Gregory Grossman. Review: Against bourgeois economic pseudo-theories of socialism. *The American Economic Review*, 53(1):211–213, 1963.

[23] KE Iverson. *A programming language*. Wiley, New York, 1966.

[24] Kenneth E. Iverson. Notation as a tool of thought. In *ACM Turing award lectures*, page 1979. ACM, New York, NY, USA, 2007.

[25] LV Kantorovich. Mathematical Methods of Organizing and Planning Production. *Management Science*, 6(4):366–422, 1960.

[26] L.V. Kantorovich. *The Best Use of Economic Resources*. Harvard University Press, 1965.

[27] N. Karmarkar. Anew polynomial-time algorithm for linear programming. *Combinatorica*, 4(4):373–395, 1984.

[28] L. Khachian. A polynomial algorithm in linear programming. *Soviet Mathematics Doklady*, 20:191–194, 1979.

[29] V. Klee and G. Minty. How good is the simplex algorithm. In O. Shisha, editor, *Inequalities-III*, pages 159–175. Academic Press, 1972.

[30] H.D. Kurz and N. Salvadori. VON NEUMANN'S GROWTH MODEL AND THE'CLASSICAL'TRADITION. *Understanding" classical" Economics: Studies in Long-Period Theory*, 1998.

[31] Oscar Lange. *On the Economic Theory of Socialism*. University of Minnesota Press, 1938.

[32] B. Lumpkin. *Geometry Activities from Many Cultures*. Walch Publishing, 1997.

[33] K. Marx. Marginal Notes to the Programme of the German Workers' Party [Critique of the Gotha Programme]. *Marx and Engels Selected Works*, 3, 1970.

[34] Karl Marx. The difference between the democritean and epicurean philosophy of nature. In *Marx-Engels Collected Works Volume 1*. Progress Publishers, 1841.

[35] Karl Marx. *Capital*, volume 1. Progress Publishers, Moscow, 1954. Original English edition published in 1887.

[36] S. Meikle. *Aristotle's Economic Thought*. Oxford University Press, 1997.

[37] S.M. Menshikov. Topicality of Kantorovich's Economic Model. *Journal of Mathematical Sciences*, 133(4):1391–1397, 2006.

[38] G. Michaelson, W. P. Cockshott, and A. F. Cottrell. Testing marx: some new results from uk data. *Capital and Class*, pages 103–129, 1995.

[39] J. Neumann. A Model of General Economic Equilibrium. *Review of Economic Studies*, 13(33):1–9, 1945.

[40] O. Neurath. Economic plan and calculation in kind. *Otto Neurath: Economic Writings 1904-1945*, 2004.

[41] Otto Neurath. The Conceptual Structure of Economic Theory and its Foundations. In Thomas Uebel and Robert Cohen, editors, *Economic Writings*. Kluwer, (1917) 2004.

[42] Otto Neurath. Economics in Kind, Calculation in Kind and their Relation to War Economics. In Thomas Uebel and Robert Cohen, editors, *Economic Writings*. Kluwer, (1919) 2004.

[43] Alex Nove. *The Economics of Feasible Socialism*. George Allen and Unwin, London, 1983.

[44] J. O'Neill. Markets, Socialism, and Information: A Reformulation of a Marxian Objection to the Market. *Social Philosophy and Policy*, 6(2):200–210, 1989.

[45] A. Peters. *Computersozialismus:Gespräche mit Konrad Zuse*. Vaduz, 2000.

[46] Arno Peters. *Das Äquivalenzprinzip als Grundlage der Global Ökonomie*. Akademische Verlagsanstalt, 1996.

[47] S. Pinker. *The Language Instinct: How the Mind Creates Language*. HarperCollins, 2000.

[48] R. Remak. Kann die Volkswirtschaftslehre eine exakte Wissenschaft werden. *Jahrbücher für Nationalökonomie und Statistik*, 131:703–735, 1929.

[49] RL Rivest, A. Shamir, and L. Adleman. A Method for Obtaining Digital Signatures and Public-Key Cryptosystems. *Communications*, 1978.

[50] L.D. Ryabev and Y.N. Smirnov. The Atomic Project, Science, and the Atomic Industry. *Atomic Energy*, 99(2):519–527, 2005.

[51] A. M. Shaikh. The empirical strength of the labour theory of value. In R. Bellofiore, editor, *Marxian Economics: A Reappraisal*, volume 2, pages 225–251. Macmillan, 1998.

[52] Piero Sraffa. *Production of commodities by means of commodities*. Cambridge University Press, Cambridge, 1960.

[53] A. M. Turing. Computing Machinery and Intelligence. *Mind*, (49):433–460, 1950.

[54] A. M. Turing. Lecture on the Automatic Computing Engine, 1947 . In B. J. Copeland, editor, *The Essential Turing*. OUP, 2004.

[55] Alan Turing. On Computable Numbers, With an Application to the Entscheidungsproblem. *Proceedings of the London Mathematical Society*, 42:230–65, 1937.

[56] Thomas Uebel. Incommensurability, Ecology,and Planning: Neurath in the Socialist Calculation Debate, 1919–1928. *History of Political Economy*, pages 309–342, 2005.

[57] CJ Van Rijsbergen. *The Geometry of Information Retrieval*. Cambridge University Press, 2004.

[58] L. von Mises. Economic calculation in the socialist commonwealth. In F A Hayek, editor, *Collectivist Economic Planning*. Routledge and Kegan Paul, London, 1935.

[59] L. Von Mises. *Socialism: An Economic and Sociological Analysis*. Johnathan Cape, 1951.

[60] Ludvig von Mises. *Human Action*. Hodge and Company, London, 1949.

[61] J. von Neumann. Mathematical Foundations of Quantum Mechanics, Engl. transl. of the 1931 German edition by RT Beyer, 1955.

[62] D. Widdows. *Geometry and Meaning. Number 172 in CSLI Lecture Notes*. University of Chicago Press, 2004.

[63] David Zachariah. Testing the labor theory of value in sweden. http://reality.gn.apc.org/econ/DZ_article1.pdf, 2004.

[64] David Zachariah. Labour value and equalisation of profit rates. *Indian Development Review*, 4(1):1–21, 2006.