APPLICATIONS OF CONTROL THEORY

APPLICATIONS DE LA THÉORIE DU CONTROLE

APPLICATION OF CONTROL THEORY TO POPULATION POLICY

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Summary

Population policy for a single nation is considered as an optimal control problem. It is studied how the population of a country like The Netherlands could be reduced from its present size and age distribution to a prescribed, stationary size and age distribution in the shortest time possible. The control variable is the annual number of live births. Two constraints are taken into account: a socio-psychological constraint consisting of a (time-dependent) lower bound on fertility, and an economic constraint in the form of an upper bound on the demographic burden. The possible effects of emigration are also studied. The problem is solved by linear programming. Numerical results that apply to The Netherlands are shown and extensively discussed.

Introduction

During recent years questions of long-term population policy on national and global scales have been extensively discussed in the popular and scientific press. Much emphasis has been placed on the need to stop unlimited population growth, and, indeed, to reverse the trend. A notable event was the appearance in Great-Britain in 1972 of the report "A Blueprint for Survival" [1]. This publication closely associates the quality of life with population density. In the report, the desirable population density is among other things derived from the food production capacity of any given area. It is concluded that the ideal population size for Great-Britain is about 30 million (as compared to a present population size of about 56 million). The Dutch version of the report [2] quotes an ideal population size of about 5 million or less for The Netherlands (as compared to a present population size of about 13.5 million). This ideal population size should be reached in the next 150 to 200 years.

A peak in the discussions around population problems was reached in 1974, when the World Population Conference took place in Bucharest. In the same year, Mesarovic and Pestel published the Second Report of the Club of Rome [3]. In this report, various scenarios for the future of the world are analyzed. An assumption of several of these scenarios is that the fertility of all regions in the model under consideration reaches a steady-state value within 35 years and remains constant thereafter. If this process would start in 1975, a steady-state condition would be reached after about 75 years, with constant population sizes and age distributions.

It is the purpose of the present investigation to study the question how much time is minimally needed for the population of a given country (in this case The Netherlands) to reach a stationary population of prescribed size. The question is formulated as an optimal control problem. To ensure that the solutions found are reasonably realistic, various constraints are imposed. The first constraint, termed the socio-psychological constraint, imposes a bound on the rate and extent to which the fertility of the population is allowed to decrease. The second constraint, referred to as the economic constraint, prevents the so-called demographic burden (also called the dependence) of the population from exceeding a prescribed bound. The demographic burden is given by the ratio of the number of individuals not of working age (in the age groups 0 to 20, and 65 and over) to the remaining individuals (in the age group 20 to 65).The demographic burden is a rough indication of the economic load imposed on the working population by the dependent part of the population.

A simple discrete-time model for the population process is developed. The control variable is the annual number of births. The method of solution is linear programming. The effects of migration (both emigration and temporary labor) will be considered. A modest sensitivity study is included as well. An extensive discussion of the results of the computations, which have all been done for The Netherlands, concludes the paper.
The present paper is a follow-up of a sequence of research reports [4], [5], [6], and a publication in Dutch [7]. The previous publication does not contain the more detailed mathematical information given in the present paper, and moreover lacks the computations and discussions of the effects of migration. A related publication considers the problem as an optimal control problem for a distributed-parameter system [8].

The demographic model

The basic demographic model is very simple. Because of the specific function woman has in the human reproductive process, we only account for the female population, which is not unusual in demographic studies. If in the sequel the total population is mentioned, it is assumed for simplicity that there are equally many men as women. Strictly speaking this is not entirely correct: in 1973 there were 993 men per 1000 women in The Netherlands [9].

We define the quantity $p(i,j)$ as the number of women in the age group from $(i-1)h$ to ih at the instant t_o+jh, where $i = 1,2,\ldots,\frac{100}{h}$, and $j = 0,1,2,\ldots\ldots$ Here h is a basic time interval, which in demographic calculations usually is 1 year. In the present calculations h has been taken 5 years, to reduce the computational load. For the instant t_o we chose January 1, 1972, 0 hours.

The basic equation of the demographic model is

$$p(i+1,j+1) = p(i,j) - \mu(i,j)p(i,j), \tag{1}$$

with $i = 1,2,\ldots,\frac{100}{h}-1$, and $j = 0,1,2,\ldots$. The first term on the right-hand side expresses that the population ages by h years during a time period of h years. The second term represents the decrease by death of the number of women in the age group from $(i-1)h$ to ih during a period of h years; $\mu(i,j)$ is a mortality coefficient, which depends both on the age group i and the time period j. The values of the mortality coefficients were determined from projections for The Netherlands for the period 1980-1999 [10], [11]. For simplicity it has been assumed that the mortality coefficients do not depend on time (hence are independent of j) for the entire time periods involved in the computations.

The equation (1) has to be supplemented with the equation

$$p(1,j+1) = u(j), \tag{2}$$

for $j = 0,1,\ldots$. Here $u(j)$ is the number of girls born during the period from t_o+jh to $t_o+(j+1)h$ and surviving at the end of this period. We shall consider $u(j)$, $j = 0,1,\ldots$, as the control variable for the problem.

It is very easy to solve the equations (1) and (2). It follows by repeated substitution

$$p(i,j) = \begin{cases} \beta(i,j)p(i-j,0), & j = 0,1,\ldots,i-1, \\ \beta(i,j)u(j-i), & j = i, i+1,\ldots, \end{cases} \tag{3}$$

for $i = 1,2,\ldots,n$, with $n = \frac{100}{h}$, and

$$\beta(i,j) = \prod_{k=1}^{\min(j,i-1)} [1-\mu(i-k,j-k)], \tag{4}$$

for $i = 1,2,\ldots,n$, and $j = 0,1,\ldots$. Here we adopt the convention that a repeated product equals 1 if the lower limit exceeds the upper limit.

Stationary population

If the mortality coefficients $\mu(i,j)$ are assumed to be independent of the time period j, and are therefore replaced with $\mu(i)$, it follows from (3) and (4) that for $j = n+1, n+2, \ldots,$

$$p(i,j) = \bar{\beta}(i)u(j-i), \qquad i = 1,2,\ldots,n, \tag{5}$$

where

$$\bar{\beta}(i) = \prod_{k=1}^{i-1} [1-\mu(i-k)], \qquad i = 1,2,\ldots,n. \tag{6}$$

The coefficient $\bar{\beta}(i)$ has a simple interpretation: it indicates the fraction of the girls born in any time period that survives after ih years. Eq. (5) shows that if the birth volumes $u(j-n), u(j-n+1),\ldots,u(j-1)$ are constant, say equal to the constant $\bar{u}$, the age distribution at time t_0+jh is given by

$$p(i,j) = \bar{\beta}(i)\bar{u}, \qquad i = 1,2,\ldots,n. \tag{7}$$

This age distribution is independent of time, and is called a stationary age distribution. Its shape is entirely determined by the coefficients $\bar{\beta}(i)$, $i = 1,2,\ldots,n$. The corresponding total size of the female population $\bar{P}$ is also independent of j; it is given by

$$\bar{P} = [\sum_{i=1}^{n} \bar{\beta}(i)]\bar{u}. \tag{8}$$

For a given total stationary population size $\bar{P}$, the corresponding stationary birth volume $\bar{u}$ may be found from (8).

Figure 1 gives a comparison of the age distribution of the female population of The Netherlands on January 1, 1972 [11], and the stationary age distribution corresponding to a total female population of 5 million. The obvious differences are accentuated by the data summarized in Table 1. In the stationary situation, the percentage of the young (age group 0 to 20) is much smaller than at present, whereas the percentage of the old (over 65) is considerably higher. The average age shifts from about 34 at present to 41 for the stationary population. These numbers illustrate that a society with a stationary age distribution will be quite different from the present.

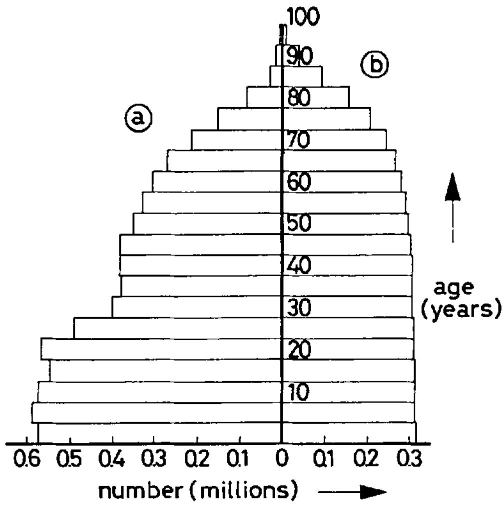

Fig. 1. (a) Age distribution of the Dutch female population on Januari 1, 1972.
(b) Stationary age distribution of the female Dutch population corresponding to a total female population size of 5 million.

TABLE 1: COMPARISON OF SOME DATA CONCERNING THE AGE DISTRIBUTION OF THE DUTCH POPULATION ON JANUARI 1, 1972, AND THE STATIONARY DISTRIBUTION

	1972	stationary
percentage women 0-20 years (%)	34.5	25.1
percentage women 20-65 years (%)	53.9	54.4
percentage women 65+ (%)	11.6	20.5
average age of women (years)	33.7	41.0

Population policy as an optimal control problem

In this section we describe how population policy may be approached as an optimal control problem. It is assumed that a population policy is to be designed that has as its goal to achieve a stationary population, of specified size, in the shortest time possible.

If no additional constraints are imposed, the solution to this problem is easily found Suppose that $\bar{P}$ is the desired stationary population size. Then from (8) we can obtain the corresponding stationary birth volume $\bar{u}$. The stationary age distribution is reached if and only if the birth volume equals $\bar{u}$ during n periods of h years, i.e., during 100 years, preceding the instant at which the stationary age distribution is achieved. Therefore, the minimum time required to reach the specified final age distribution from an arbitrary initial age distribution is nh = 100 years, except in the unlikely case that the birth volume has equalled the stationary volume $\bar{u}$ during a certain length of time before the initial time t_o. This case will not be considered.

We thus conclude that the minimum time in which a stationary population may be reached is 100 years.The size of this stationary population may be arbitrarily specified. If the target population size is very small (say, 3 million as compared to the present 6.6 million women), the transition from the present age distribution to the terminal distribution will show various undesirable phenomena. First of all, it is to be expected that the birth rate, defined here as the annual number of female births per 1000 females in the fertile age, will temporarily drop to extremely low values during the first decades. Secondly, there will be a period (later than the first-mentioned period).during which the population in the age group 65+ has a very large size as compared to the working population (age group 20 to 65), thus imposing an unadmissible large economic burden on the working population.

To prevent these effects, constraints will be imposed on the solution, which will be discussed in the next sections. The purpose of these constraints is to find more realistic population planning programs,which have some chance of being implementable.

Socio-psychological constraint

An important element in projections of population growth is the so-called fertility pattern. The fertility pattern describes the age specific fertility of women. Figure 2 represents the fertility pattern that was observed in The Netherlands in 1969. The plot shows for each five-year age group (10 to 15, 15 to 20, etc.) the average annual number of surviving girls born during a future period of 5 years from 1000 women in the relevant age group.

We shall assume - in common with the Second Report to the Club of Rome [3] - that the shape of the fertility pattern does not vary with time, but that the pattern may decrease or increase as a whole. We shall furthermore assume that for each projected time period there exists a fertility pattern that imposes a lower bound on the birth

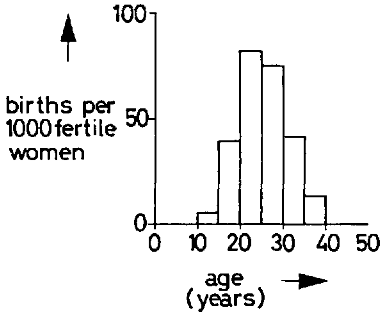

Fig. 2. Fertility pattern observed in The Netherlands in 1969.

volumes. This means that the actual fertility pattern always has to exceed the assumed minimum pattern. Thus we suppose that during the time period from $t_o+(j-1)h$ to t_o+jh the annual number of female births per 1000 women in the age group from $(i-1)h$ will at least have to equal $m(i,j)$. This number is considered as the least socially acceptable number for the relevant time period. Since the total number of births is obtained by summing the numbers of births from women in the various age groups, this socio-psychological constraint imposes the following restriction on the number of births:

$$u(j) > \sum_{i=1}^{n} hm(i,j)p(i,j)/1000 \tag{9}$$

for $j = 0,1,2,\ldots$.

In the context of this study a certain choice was made for the behavior of the minimal fertility pattern. We assume an exponential decrease from the initial pattern. The initial pattern is taken to be 5% below the pattern observed in 1969. The dependence of the pattern on time is given by

$$m(i,j) = [r + (1-r)e^{-jh/\theta}]m(i,0), \tag{10}$$

$j = 0,1,2,\ldots,$ where θ is a time constant, and r the fraction of the initial pattern to which the pattern is eventually reduced. In the calculations, unless stated otherwise, we have taken $r = 0.6$, and $\theta = 20$ years. This means that minimum fertility is reduced to 60% of the initial value over a time period of about 40 years.

It is to be expected that the sensitivity of the solution to variations in r and θ is relatively great. A simple sensitivity study is presented in a later section.

With the introduction of the side-condition (9), we have to consider the problem of finding $u(j)$, $j = 0,1,2,\ldots,N-1$, as well as N, such that N is minimal, while (9) is satisfied for $j = 0,1,\ldots,N-1$, and

$$p(i,N) = \bar{p}(i), \qquad i = 1,2,\ldots,n. \tag{11}$$

Here $\bar{p}(i)$, $i = 1,2,\ldots,n$, is the age distribution corresponding to the desired stationary population, with prescribed size $\bar{P}$.

This optimal control problem is a minimum-time problem. Since the solution of minimum-time problems, especially in the discrete-time case, involves certain complications, we prefer to solve a related problem, whose solution yields the answer to the original problem. Therefore, we consider the problem of finding, for given N, the minimum size of the stationary population that may be reached at time N, while satisfying the socio-psychological constraint. Thus we have to find $u(j)$, $j = 0,1,2,\ldots,N-1$, with N given, such that (9) is satisfied for $j = 0,1,\ldots,N-1$, such that $p(i,N)$, $i = 1,2,\ldots,n$, is a stationary age distribution, and such that

$$P = \sum_{i=1}^{n} p(i,N) \tag{12}$$

is minimal. Suppose that this problem has been solved, and let $P_{min}(N)$ indicate the minimum stationary population size reachable within N time periods. It will be seen, and indeed is very plausible, that P_{min} is a strictly decreasing function of N. Therefore, once we have a plot of P_{min} as a function of N, it is very easy to determine the minimum number of time periods N necessary to reach a given stationary population size $\bar{P}$.

We now discuss the solution of the second problem described. The age distribution at time $N \geq n$ is stationary if and only if $u(j) = \bar{u}$, with $\bar{u}$ a constant to be determined, for $j = N-n, N-n+1,\ldots,N-1$. Then we have

$$p(i,N) = \bar{\beta}(i)\bar{u}, \qquad \sum_{i=1}^{n} p(i,N) = [\sum_{i=1}^{n} \bar{\beta}(i)]\bar{u}. \tag{13}$$

Hence, (12) is minimized if $\bar{u}$ is minimized. Substitution of (3) into the constraint (9) yields

$$u(j) \geq \sum_{i=1}^{\min(j,n)} 1000hm(i,j)\beta(i,j)u(j-i) + \sum_{i=j+1}^{n} 1000hm(i,j)\beta(i,j)p(i-j,0),$$

$$j = 0,1,\ldots,N-1. \tag{14}$$

We adopt the convention that a summation cancels if the lower limit exceeds the upper limit.

Thus we have to solve the following problem: minimize $\bar{u}$ with respect to the independent variables $u(j) \geq 0$, $j = 0,1,\ldots,N-1$, subject to $u(N-n) = u(N-n+1) = \ldots\ldots = u(N-1) = \bar{u}$ and subject to (14). This is a straightforward linear programming problem, which is easily solved numerically once a standard code is available.

Fig. 3 gives some of the numerical results. The solid curve represents the minimal stationary female population size as a function of the time needed to reach it. The plot shows that the minimum female population size reachable in 100 years - the minimal time needed to reach a stationary population of any size - is 9.47 million, corresponding to a total population size of about 18.9 million. The curve also shows that the time needed to reach a stationary total population size of 5 million (2.5 million women) - the ideal population size quoted in the Dutch version of "A Blueprint for Survival" [2] - is about 220 years.

Fig. 3 also indicates - with dashed lines - the time histories of the total population sizes eventually reaching stationary female population sizes of respectively 9.47 million,5.90 million,and 3.21 million. The time periods required successively are 100, 150, and 200 years.

Fig. 4 shows how the birth volumes would have to behave to reach stationary female populations of respectively 9.47 million, 5.90 million and 3.21 million. The following pattern emerges. Initially the birth volume precisely equals the minimum value allowed by the psycho-sociological constraint. This continues until the instant at which the psycho-sociological minimum value equals the stationary birth volume corresponding to the desired stationary population size. From this instant on it takes 100 years until the stationary situation is reached.

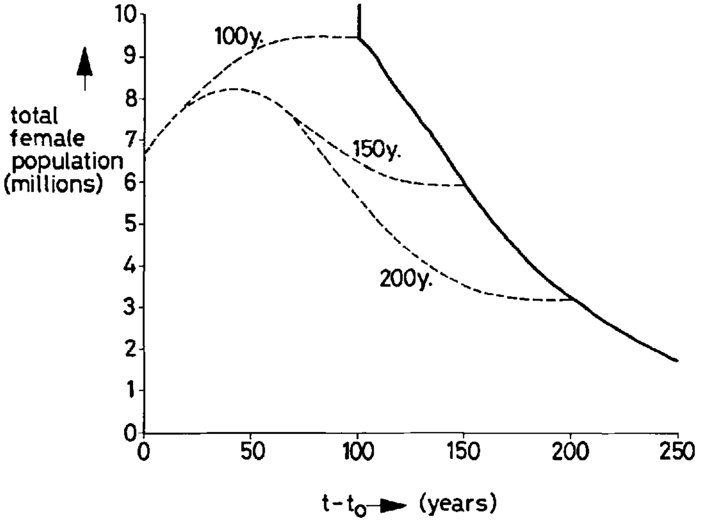

Fig. 3. Total population size as a function of time; socio-psychological constraint only.

Additional clarification is provided in Fig. 5. Here we show for each of the three cases considered how fertility behaves with time. Fertility was defined as the annual number of female births per 1000 fertile women. Fertile women by definition are women in the age group 15 to 40. In each case fertility eventually stabilizes at the value of 40.6, which is the value needed to maintain a stationary population. To achieve an eventual reduction in population size (5.90 million respectively 3.21 million as compared to the initial 6.6 million), fertility temporarily has to assume values below the equilibrium fertility.

A guideline for a practical population policy that has as its goal to achieve a stationary population of prescribed size evidently is first to reduce fertility as quickly as socially possible, and then slowly let it increase again to the equilibrium value.

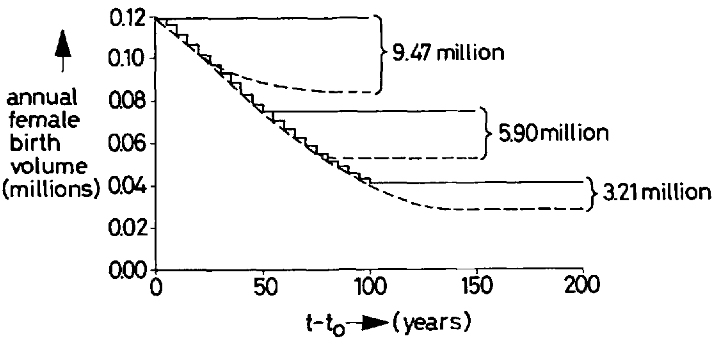

Fig. 4. Annual female birth volumes for different target populations. The dashed lines indicate the minimal socially acceptable birth volumes. Socio-psychological constraint only.

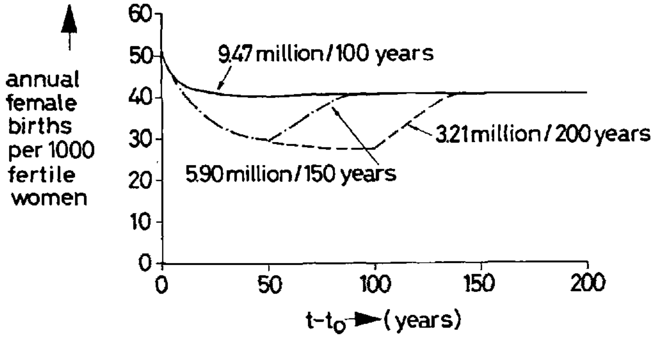

Fig. 5. Fertility as a function of time, for three different target populations. Socio-psychological constraint only.

Demographic burden; economic constraint

An important indicator for demographic processes is the demographic burden as defined in the Introduction. The demographic burden gives a rough indication of the economic effort the working part of the population has to make for the dependent, non-working part. On January 1, 1972, the demographic burden for the female Dutch population [9] was 0.856; for the total population it was 0.841. This last number means that each person in the age group from 20 to 65 has to take care of 0.841 person in the dependent age groups, as well as of himself.

Fig. 6 exhibits plots of the time histories of the demographic burden for each of the three cases considered in the previous section. These time histories typically show three periods. In the first period the demographic burden decreases considerably due to the diminishing juvenile part of the population. This trend reverses soon, until the demographic burden reaches a relatively high value in the second period, which may be attributed to the relative increase of the size of the age group over 65. Following this, the demographic burden settles at its stationary value of 0.837 for the female population.

The peak value that is reached by the demographic burden is about 0.95 for the third time history. Although it is not clear whether this value is insupportably high, it is of some interest to see what happens if the demographic burden is constrained not to exceed a given maximum value. The demographic burden of the female population during the time period j is given by

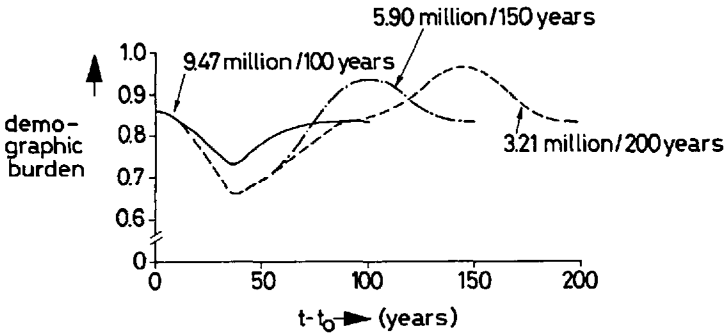

Fig. 6. Demographic burden as a function of time, for three different target populations. Socio-psychological constraint only.

$$d(j) = \frac{\sum_{i \in W^c} p(i,j)}{\sum_{i \in W} p(i,j)}, \tag{15}$$

where $W = \{\frac{20}{h}+1, \frac{20}{h}+2, \ldots, \frac{65}{h}\}$, and W^c is the complement of W in $\{1,2,\ldots,n\}$.

We now add the economic constraint

$$d(j) \leq \alpha \quad , \quad j = 0,1,\ldots,N, \tag{16}$$

where α is a prescribed maximum value. Using (15), this can be rewritten as

$$\sum_{i \in W^c} p(i,j) \leq \alpha \sum_{i \in W} p(i,j), \qquad j = 0,1,\ldots,N. \tag{17}$$

Substitution of p(i,j) as given by (3) adds another set of inequality constraints to the linear constraints of the linear program described in the previous section. Again, numerical solution is straightforward if a linear programming code is available.

Table 2 gives some of the results, where the maximum demographic pressure α is rather arbitrarily taken to be 0.9, slightly higher than the present value of 0.856. The minimal population achievable in a given time span is slightly higher than in the case without economic constraint, but the differences are not alarming. The plots of Fig. 7 indicate the time histories of the birth volumes in comparison with the corresponding cases without economic constraint. It turns out that the time histories only undergo modifications around the time that the birth volume makes its transition from the minimum value to the stationary value. Fig. 8 shows plots of the demographic burden as a function of time in case the economic constraint is imposed. It is seen that the general pattern remains approximately the same, except that the peaks over 0.9 are cut off.

TABLE 2: RELATION BETWEEN POPULATION SIZE AND TIME SPAN REQUIRED TO REACH IT

time span (years)	female population size achieved <u>without</u> economic constraint (millions)	female population size achieved <u>with</u> economic constraint (millions)
100	9.47	9.47
150	5.90	6.30
200	3.21	3.77

On the whole, the effect of the economic constraint is minor. Moreover, it is to be expected that the problems caused by a demographic burden that is temporarily too high can be eased by temporary immigration (guest workers). We shall therefore omit the economic constraint in the remaining discussions.

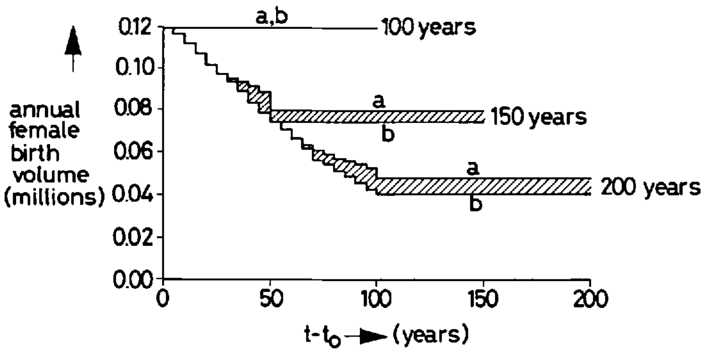

Fig. 7. Annual female birth volumes for three different time spans. a: economic and socio-psychological constraint; b: psycho-sociological constraint only.

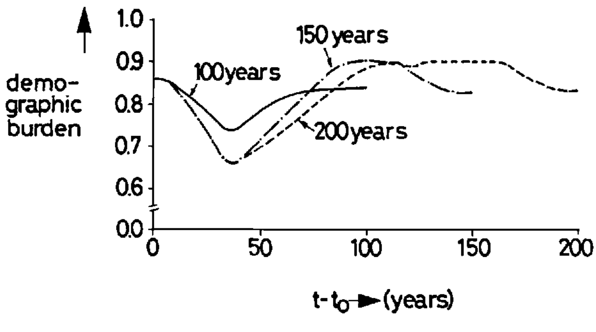

Fig. 8. Demographic burden as a function of time with economic constraint for three different time spans.

Migration

Up to this point we have totally ignored the possible effects of migration. It is clear that if the population size is to be reduced, emigration may play a useful role. At the end of the preceding section we have noted that temporary immigration may help to overcome periods of high demographic burden. As temporary immigration has no lasting effect on the population evolution and structure, in this section we shall only consider migration, and in particular emigration, of the autochtonous population.

To account for emigration, the basic demographic model (1) has to be modified to

$$p(i+1,j+1) = [1-\mu(i,j)]p(i,j)-M(i,j), \tag{18}$$

$i = 1,2,\ldots,n-1$; $j = 0,1,\ldots$. Here $M(i,j)$ is the number of females who at time t_o+jh are in the age group from $(i-1)h$ to ih and who emigrate during the period from t_o+jh to $t_o+(j+1)h$. Solution of these equations together with (2) yields

$$p(i,j) = \begin{cases} \beta(i,j)p(i-j,0) - \sum_{k=1}^{\min(i-1,j)} \beta_k(i,j)M(i-k,j-k), & j = 0,1,\ldots,i-1, \\ \beta(i,j)u(j-i) - \sum_{k=1}^{\min(i-1,j)} \beta_k(i,j)M(i-k,j-k), & j = i, i+1,\ldots, \end{cases} \tag{19}$$

where $\beta(i,j)$ is as defined before, and

$$\beta_k(i,j) = \prod_{s=1}^{k-1} [1-\mu(i-s,j-s)], \tag{20}$$

$i = 1,2,\ldots n$; $j = 0,1,2,\ldots$; $k = 1,2,\ldots,\min(i-1,j)$.

For lack of more detailed information, we assume that the age distribution of the emigrating population does not vary with time, so that

$$M(i,j) = q(i)e(j), \quad i = 1,2,\ldots,n, \ j = 0,1,\ldots. \tag{21}$$

Here $q(i)$ is the fraction of emigrating women in the age group from $(i-1)h$ to ih, $i = 1,2,\ldots n$, while $e(j)$ represents the total number of women emigrating during the period t_o+jh to $t_o+(j+1)h$.

The choice of the age distribution $q(i)$, $i = 1,2,\ldots n$, was made on the basis of emigration data for The Netherlands during the period 1950 to 1953 [12]. This was a period of high emigration. The distribution is graphically presented in Fig. 9.

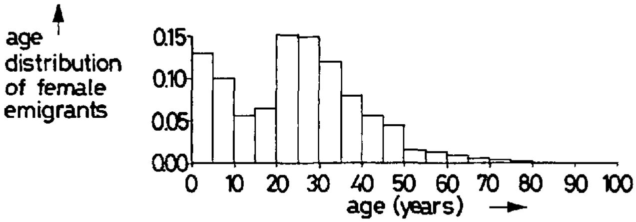

Fig. 9. Age distribution of female emigrants. Fraction of female emigrants in each age group.

The emigration volumes $e(j)$, $j = 0,1,\ldots,N-1$, are control variables, in addition to the birth volumes $u(j)$, $j = 0,1,\ldots,N-1$. In order to achieve a stationary situation - without emigration - at the final time N, emigration has to be stopped during the n time periods before the final instant. We therefore set $e(N-n) = e(N-n+1) = \ldots = e(N-1) = 0$, which leaves $e(j)$, $j = 0,1,\ldots,N-n-1$, as control variables. An additional benefit of this is that there is no emigration during periods of high demographic burden (see the plots of Fig. 6), which appears very reasonable.

Emigration is constrained by the requirement

$$0 \leq e(j) \leq \xi(j)h \sum_{i=1}^{n} p(i,j), \quad j = 0,1,\ldots,N-n-1. \tag{22}$$

Here $\xi(j)$ is the maximal fraction of the total female population annually emigrating during the period from t_o+jh to $t_o+(j+1)h$.

We can now consider the problem of minimizing the final stationary population size, including the contribution of emigration, while taking into account the socio-psychological constraint, the economic constraint, or both. Since the economic constraint was seen to be of minor importance, first only the socio-psychological constraint is included. Substitution of $p(i,j)$ as given by (19) into (9) and (22) leads to another linear programming problem, with a number of inequality constraints, and with $u(j)$, $j = 0,1,\ldots,N-n-1$, $\bar{u}$, and $m(j)$, $j = 0,1,\ldots,N-n-1$, as independent variables. Also this problem can be solved using a standard linear programming code together with an input program to set up the initial tableau.

Numerical results were obtained for $\xi(j) = 0.05$, $j = 0,1,\ldots,N-n-1$, i.e. a maximal emigration of 0.5% annually. This figure corresponds to the highest percentage observed in the fifties. Figure 10 gives some of the results for a total period of 150 years. Emigration is always at its maximal value. The annual birth volume steadily decreases until after about 35 years, when it exhibits a steep rise before falling off to its steady-state value. The peak may be explained by the interference of the social and the emigration constraint: in order to allow emigration to assume its maximal

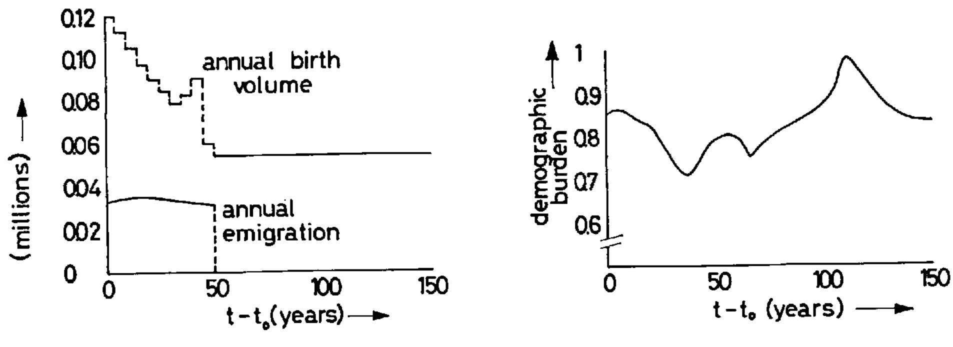

Fig. 10. Annual female births, emigration and demographic burden for a time span of 150 years. Socio-psychological and emigration constraints only.

value, the birth volume temporarily has to rise.

The right-hand side of Fig. 10 shows the time behavior of the demographic burden corresponding to the left-hand side. The irregular behavior may be attributed to the strongly varying behavior of the birth volume. The peak value of 0.98 is higher than in the case without emigration.

The total stationary female population size that is reached after 150 years is 4.30 million, as compared to 5.90 million without emigration. This shows that the potential effect of emigration is considerable.

In a subsequent computation, of which no graphs are shown, the demographic burden was constrained not to exceed the value 0.9. The solution features maximal emigration, a less irregular behavior of the birth volume, and a final stationary female population size of 4.59 million.

Sensitivity study

A simple sensitivity study has been performed by repeating some of the calculations while varying the most critical parameters. In the computations involving the socio-psychological constraint, the parameters r (the eventual maximal reduction in fertility) and θ (the time constant corresponding to which fertility decreases) were initially rather arbitrarily chosen and therefore open to question. Table 3 shows results of computations for a total period of 150 years and different values of r and θ. It is seen that the differences in eventual population sizes are considerable, which is not unexpected.

TABLE 3. CALCULATIONS WITH SOCIO-PSYCHOLOGICAL CONSTRAINT FOR A TIME PERIOD OF 150 YEARS. EVENTUAL FEMALE POPULATION SIZES FOR DIFFERENT VALUES OF r AND θ (MILLIONS).

	r		
θ (years)	0.5	0.6	0.7
10	3.53	4.91	6.52
20	4.58	5.90	7.38
30	5.60	6.81	8.14

Discussion

The results of the computations that have been made in the context of this study lead to a number of conclusions. First of all we may conclude that if the goal of a population policy for The Netherlands is to reduce the present size of 13.5 million to a stationary size of 5 million or less - the objective of the Dutch version of "A Blueprint for Survival" [2] - this goal cannot be reached within 200 years (without emigration). Furthermore, it is found that the minimum time to reach a more or less stationary population - no matter what size - is about 80 years; the resulting total population size would be about 19 million.

The computations also show that (without emigration) the social constraint essentially determines the time needed to reach a desired population size. The economic constraint, included to prevent anomalies in the age distribution, turns out to play a minor role. Emigration may be a helpful factor in reducing the eventual population size.

It may be furthermore be seen from the numerical results that the long-term goal of reaching a stationary population of reduced size for the next decades may be translated into short-term tactics consisting of reducing fertility as quickly as socially acceptable - not a very surprising result. The time period over which this short-term policy is to be continued depends on the desired eventual population size and may vary from 50 to 150 years. This means that a decision concerning the desired population size may be postponed for some time.

There is no need to discuss extensively the means that are available for the implementation of a population policy as outlined. Important instruments are: information about birth control, education of the public, social security policy, tax measures, and abortion legislature. The use of an active emigration policy is also evident.

There is no question that the transition as studied in this paper, from the present society to a society with a stationary age distribution and a reduced size, will have major effects. The sweeping changes in age distribution and population size will have far-reaching consequences for the economic activity, education, health care, housing and social security [13]. To illustrate this, Figure 11 shows the time behavior of two indicators for the transition requiring 150 years (without economic constraint and without emigration). The figure presents the time history of the numbers of women in the age groups 0-20 and 65+, in absolute magnitudes and as percentages of the total female population. It is seen that the age group 0-20 after an initial slight increase steadily decreases in size, to stabilize after about 70 years at a constant value. This phenomenon will of course sharply affect the requirements for educational facilities.

The age group 65+ on the other hand increases to more than double its present size. The maximum is reached in about 75 years, after which this population group starts decreasing again. This effect will have important consequences for the need for all sorts of provisions for the aged.

Of course the credibility of the results of the computations strongly depends on the explicit and implicit assumptions. In the sensitivity analysis of the preceding section the effects of changing certain assumptions are pointed out. All of the qualitative conclusions, as presented in the preceding paragraphs, remain unaffected, however.

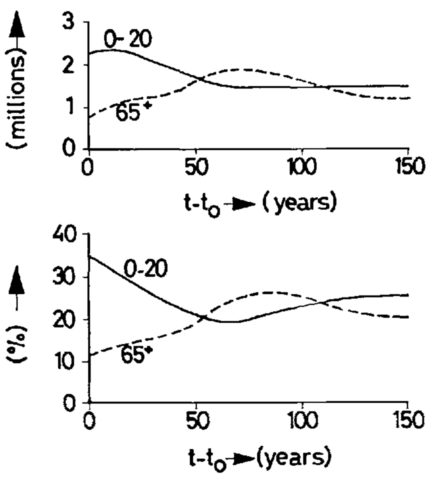

Fig. 11. Time histories of the sizes of the age groups 0-20 and 65+. Socio-psychological constraint only; time span 150 years.

Conclusion

The purpose of the study reported in this paper was to investigate whether considering population policy as an optimal control problem could help determining the possibilities and impossibilities of practical population policy. It is seen that interesting conclusions can be reached, both quantitatively and qualitatively.

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